



螺旋線 helix $X = [a \cos \theta, a \sin \theta, b\theta]$

螺旋面 helicoid

$$X = [r \cos \theta, r \sin \theta, a\theta + b]$$

$$X_r = [\cos \theta, \sin \theta, 0]$$

$$X_\theta = [-r \sin \theta, r \cos \theta, a]$$

$$\text{則 } N = \frac{X_r \times X_\theta}{|X_r \times X_\theta|} = [a \sin \theta, -a \cos \theta, r]$$

$$E = F = G = e = f = g =$$

$$H = \frac{Eg - 2Ff + Ge}{2(EG - F^2)} = 0$$

或者 $X(u, v) = (a \sinh v \cos u, a \sinh v \sin u, au)$

$$E = G = a^2 \cosh^2 v, F = 0$$

$$X_{uu} + X_{vv} = 0$$

所以 $H = 0$

§ Enneper's minimal surface

$$X(u, v) = \left(u - \frac{u^3}{3} + uv^2, v - \frac{v^3}{3} + vu^2, u^2 - v^2\right)$$

§ Circular helix (圓柱螺旋線)

A helix is defined by $t \cdot a = \cos \alpha = \text{const}$

$$\Leftrightarrow \frac{\kappa}{\tau} = \tan \alpha = \text{const}$$

設有一曲線 Γ 的曲率 κ ，扭率 τ ，滿足
$$\begin{cases} \kappa^2 + \tau^2 = 1 \\ \kappa\tau = \frac{1}{2} \end{cases}$$
，則 Γ 是圓柱螺旋線。