z=f(x,y)

X=[x,y,f(x,y)]

$X_x = [1, 0, \frac{\partial z}{\partial x}], X_y = [0, 1, \frac{\partial z}{\partial y}]$，假設 $\frac{\partial z}{\partial x} = p, \frac{\partial z}{\partial y} = q$

...則

$$ds^2 = (1+p^2)dx^2 + 2pqdxdy + (1+q^2)dy^2$$

$$N = \frac{X_x \times X_y}{\left|X_x \times X_y\right|} = \frac{[-p, -q, 1]}{\sqrt{1+p^2+q^2}}$$

$$dA = \sqrt{EG - F^2}dxdy = \sqrt{1+p^2+q^2}dxdy$$

假設

$$II = edx^2 + 2fdxdy + gdy^2 = \frac{rdx^2 + 2sdxdy + tdy^2}{\sqrt{1+p^2+q^2}}$$

其中 $r = \frac{\partial^2 z}{\partial x^2}, s = \frac{\partial^2 z}{\partial x \partial y}, t = \frac{\partial^2 z}{\partial y^2}$

習作

1. 所有 minimal surface z=f(x,y)滿足 $t(1+p^2)-2pqs+r(1+q^2)=0$

$$H=\frac{eG+gE-2fF}{2(EG-F^2)}$$

2. 試證可展曲面 z=f(x,y)的微分式為 $\frac{\partial^2 z}{\partial x^2}\cdot\frac{\partial^2 z}{\partial y^2}-(\frac{\partial^2 z}{\partial x\partial y})^2=0$

可展曲面 $\Leftrightarrow K=\frac{eg-f^2}{EG-F^2}=0$

3. 下列的微分式所代表的幾何意義為何？其中 z=f(x,y)

(1) $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=0$ ......right conoid

(2) $y\frac{\partial z}{\partial x}-x\frac{\partial z}{\partial y}=0$ ......以 z 軸為旋轉軸的旋轉面

right conoid $\begin{cases} x=r\cos\theta \\ y=r\sin\theta \\ z=f(\theta) \end{cases}$

旋轉面 $\begin{cases} x=r\cos\theta \\ y=r\sin\theta \\ z=f(r) \end{cases}$

4. 求曲面 z=f(x,y)的曲率曲線的方程式(lines of curvature)

$\begin{vmatrix} dv^2 & -dudv & du^2 \\ E & F & G \\ e & f & g \end{vmatrix}=0$，now du=dx,dv=dy

$$\frac{1}{\sqrt{1+p^2+q^2}}\begin{vmatrix} dy^2 & -dxdy & dx^2 \\ 1+q^2 & -pq & 1+p^2 \\ r & s & t \end{vmatrix}=0$$

5. 求曲面 z=f(x,y)的 Γ_{ij}^k，用 p,q,r,s,t 表示

可展曲面(developable surface)

1. Cone
2. Cylinder
3. tangent developable(the surface generated by the tagent lines to a space curve)

Cone

$X(r,\theta)=[r\cos\theta\cos\varphi, r\sin\theta\cos\varphi, r\sin\varphi]$

$X_r=[\cos\theta\cos\varphi, \sin\theta\cos\varphi, \sin\varphi]$

$X_\theta=[-r\sin\theta\cos\varphi, r\cos\theta\cos\varphi, 0]$

$N=\dfrac{X_r\times X_\theta}{|X_r\times X_\theta|}=[-\cos\theta\sin\varphi, -\sin\theta\sin\varphi, \cos\varphi]$

$X_{rr}=[0,0,0]$

$X_{r\theta}=[-\sin\theta\cos\varphi, \cos\theta\cos\varphi, 0]$

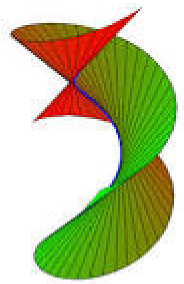
$X_{\theta\theta}=[-r\cos\theta\cos\varphi, -r\sin\theta\cos\varphi, 0]$

$e=X_{rr}\cdot N=0$

$f=X_{r\theta}\cdot N=0$

$g=X_{\theta\theta}\cdot N=r\sin\varphi\cos\varphi$

所以 $K=\dfrac{eg-f^2}{EG-F^2}=0$



The tagent developable of a helix

$X(s,v)=y(s)+vy'(s)$，其中 y 是一空間曲線

$X_s=y'+vy''=t+v\kappa n$

$X_v=y'=t$

N=b

$X_{ss}=\kappa n+v\kappa(-\kappa t+\tau b)$

$X_{sv}=\kappa n$

$X_{vv}=0$

Then $e=-v\kappa\tau, f=0, g=0$

所以 K=0