

v, w 是沿著曲線 $\alpha: I \rightarrow S$ 的向量場，試證共變微分的 Leibniz 法則

$$\text{即 } \frac{d}{dt} \langle v(t), w(t) \rangle = \left\langle \frac{Dv}{dt}, w(t) \right\rangle + \left\langle v(t), \frac{Dw}{dt} \right\rangle$$

$$\frac{d}{dt} \langle v(t), w(t) \rangle = \left\langle \frac{dv}{dt}, w(t) \right\rangle + \left\langle v(t), \frac{dw}{dt} \right\rangle$$

$v(t)$ 是沿著曲線的向量場，所以只有切部

$$\text{假設 } \frac{dw}{dt} = \left(\frac{dw}{dt}\right)^T + \left(\frac{dw}{dt}\right)^N, \text{ 則 } \left\langle \frac{dw}{dt}, v(t) \right\rangle = \left\langle \left(\frac{dw}{dt}\right)^T, v(t) \right\rangle = \left\langle \frac{Dw}{dt}, v(t) \right\rangle$$

$$\text{同理 } \left\langle \frac{dv}{dt}, w(t) \right\rangle = \left\langle \frac{Dv}{dt}, w(t) \right\rangle$$