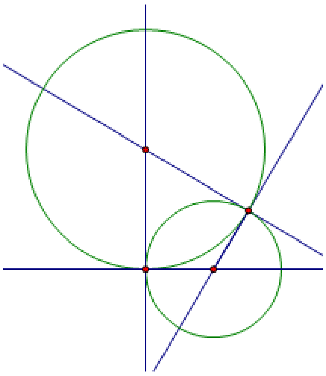

C : $r = a\cos\theta$ 是一曲線族, a 為參數

求其正交曲線族

設 $X = [r\cos\theta, r\sin\theta]$ 是平面的坐標系

則 $X_r = [\cos\theta, \sin\theta]$

$X_\theta = [-r\sin\theta, r\cos\theta]$

E=1, F=0, G=r^2

C : $dr + a\sin\theta d\theta = 0$

$dX = X_r dr + X_\theta d\theta$, 其正交曲線 $\delta X = X_r \delta r + X_\theta \delta\theta$

$dX \cdot \delta X = 0$

$dr\delta r + r^2 d\theta\delta\theta = 0$, 同除以 dr

$$\delta r + r^2 \cdot \frac{-1}{a\sin\theta}\delta\theta = 0$$

$a\sin\theta\delta r = r^2\delta\theta$, $a = \dfrac{r}{\cos\theta}$

$$\frac{\delta r}{\delta\theta} = \frac{r^2}{\frac{r}{\cos\theta}\sin\theta} = \frac{r}{\frac{\sin\theta}{\cos\theta}}$$

$$\frac{\delta r}{r} = \frac{\cos\theta\delta\theta}{\sin\theta} = \frac{\delta\sin\theta}{\sin\theta}$$

$\ln r = \ln\sin\theta + c$

$r = e^c \sin\theta = b\sin\theta$

如果取 C′ : $r = b\sin\theta$, 則 $\delta r = b\cos\theta\delta\theta$

$dX \cdot \delta X = dr\delta r + r^2 d\theta\delta\theta = (-a\sin\theta d\theta)(b\cos\theta\delta\theta) + r^2 d\theta\delta\theta$

$=0$ (因為在交點滿足 $\begin{cases} r = a\cos\theta \\ r = b\sin\theta \end{cases}$)