$z=\sinh x\sqrt{1-(\frac{y}{\cosh x})^2}$ 求 mean curvature H=

$\sinh x=\frac{e^x-e^{-x}}{2},\cosh x=\frac{e^x+e^{-x}}{2}$ ， $\cosh^2 x-\sinh^2 x=1$

$$H=\frac{eG-2fF+\text{gE}}{2(EG-F^2)}$$

$$X(x,\theta)=(x,\cosh x\sin\theta,\sinh x\cos\theta)$$
$$X_x=(1,\sinh x\sin\theta,\cosh x\cos\theta)$$
$$X_\theta=(0,\cosh x\cos\theta,-\sinh x\sin\theta)$$

$$E=X_x\cdot X_x=1+\sinh^2 x\sin^2\theta+\cosh^2 x\cos^2\theta$$

$$F=X_x\cdot X_\theta=0$$

$$G=X_\theta\cdot X_\theta=\cosh^2 x\cos^2\theta+\sinh^2 x\sin^2\theta$$

$$X_x\times X_\theta=(-\sinh^2 x-\cos^2\theta,\sinh x\sin\theta,\cosh x\cos\theta)$$

$$\left|X_x\times X_\theta\right|^2=$$

$$N=\frac{X_x\times X_\theta}{\left|X_x\times X_\theta\right|}=\frac{\cdots}{\sqrt{(\sinh^2 x+\cos^2\theta)(\sin^2 x+\cos^2\theta+1)}}$$

$$X_{xx}=(0,\cosh x\sin\theta,\sinh x\cos\theta)$$
$$X_{x\theta}=(0,\sinh x\cos\theta,-\cosh x\sin\theta)$$
$$X_{\theta\theta}=(0,-\cosh x\sin\theta,-\sinh x\cos\theta)$$

$$e=\frac{\cosh x\sinh x}{\sqrt{(\sinh^2 x+\cos^2\theta)(\sinh^2 x+\cos^2\theta+1)}}$$

$$g=\frac{-\cosh x\sinh x}{\sqrt{(\sinh^2 x+\cos^2\theta)(\sinh^2 x+\cos^2\theta+1)}}$$

$$H=\frac{eG+gE}{2EG}=\frac{e(\text{G-E})}{2EG}=\frac{-e}{2\text{EG}}$$

For fixed x， $(\frac{y}{\cosh x})^2+(\frac{z}{\sinh x})^2=\cos^2\theta+\sin^2\theta=1$

This symmetry implies the surface is rotationally invariant around the x-axis，a hallmark(標誌) of minimal surfaces like the catenoid。

The term e contains coshxsinhx in the numerator，but for a minimal surface，these terms must inherently(本質上) cancel globally due to the surface's geometric constraints

(e.g., the identity $\cosh^2 x - \sinh^2 x = 1$)
Catenoid：

$$\begin{cases} x = c\cosh(\frac{v}{c})\cos u \\ y = c\sinh(\frac{v}{c})\sin u \\ z = v \end{cases}$$

For fixed x，$(\frac{y}{\cosh x})^2 + (\frac{z}{\sinh x})^2 = \cos^2\theta + \sin^2\theta = 1$ is a catenoid。

The term e contains $\cosh x \sinh x$ in the numerator, but H must vanish ($H = 0$) for a minimal surface. This forces the **geometric cancellation** of $\cosh x \sinh x$ through two mechanisms:

1. **Hyperbolic Identity**: The identity $\cosh^2 x - \sinh^2 x = 1$ is implicitly embedded in the parameterization, ensuring that contributions from $\cosh x \sinh x$ are counterbalanced by the surface's curvature structure.
2. **Symmetry of the Parameterization**: The rotational symmetry ensures that the positive and negative contributions of $\cosh x \sinh x$ (via e and $g = -e$) cancel globally when integrated over the surface. This is a hallmark of minimal surfaces, where stretching and compression effects balance perfectly.

The geometric constraints of minimality (H=0) and the symmetry of the catenoid parameterization force the $\cosh x \sinh x$ terms to cancel identically。This cancellation is not algebraic but geometric，arising from the *surface's* intrinsic balance of curvature contributions。

Thus，H=0