§ 01 函數的 Morse index

$f(x,y)=x^2-y^2$

$\nabla f=(2x,-2y)$ 唯一臨界點為(0,0)

Hessian matrix $H_f(x,y)=\begin{pmatrix}\partial^2 f/\partial x^2 & \partial^2 f/\partial x\partial y\\ \partial^2 f/\partial x\partial y & \partial^2 f/\partial y^2\end{pmatrix}=\begin{pmatrix}2 & 0\\ 0 & -2\end{pmatrix}$

Eigenvalues are ± 2，the Morse index of the critical point =1

§ 02 CMC 曲面，計算其 Morse index

(1) $X(u,v)=(\cosh u\cos v,\cosh u\sin v,u), u\in(0,2\pi], v\in R$

幾個方向可以使其表面積變小(面積泛函 A 的二次變分 $\delta^2 A(v,v)<0$ 的方向有幾個獨立方向。)

建立自伴算子 $L\phi=-\Delta\phi-|A|^2\phi$

求該算子所有負 eigenvalue 的數量(Sturm-Liouville problem)

(2) S^2 Morse index =0

(3) Catenoid Morse index =1

(4) Scherk(II) $z=\ln(\frac{\cos x}{\cos y})$ Morse index =2

(5) Pseudosphere Morse index =1