

§ The maximum principle [\[PDE102\]](#) [\[RF003\]](#)

Second partials test: Let (a, b) be a critical point of f .

Define the quantity $d = f_{xx}(a, b)f_{yy}(a, b) - [f_{xy}(a, b)]^2$. Then, we have the following.

1. $d > 0, f_{xx}(a, b) > 0 \Rightarrow$ relative minimum.
2. $d > 0, f_{xx}(a, b) < 0 \Rightarrow$ relative maximum.
3. $d < 0, \Rightarrow$ saddle point.
4. $d = 0$: Test is inconclusive.

$$f(x, y) = 2x + 4y - x^2 - y^2$$

$$\frac{\partial f}{\partial x} = 2 - 2x, \quad \frac{\partial f}{\partial y} = 4 - 2y, \quad \text{critical point } (x, y) = (1, 2) \text{ is a maximum.}$$

$$f_{xx} = -2, \quad f_{yy} = -2, \quad f_{xy} = 0$$

$$d = f_{xx}(1, 2)f_{yy}(1, 2) - (f_{xy}(1, 2))^2 = 4 > 0, \quad f_{xx}(1, 2) < 0$$

Hence $(1, 2)$ has a relative maximum=5