§ Gradient Ricci soliton(梯度孤立子)

AGradient Ricci solitonis a special solution to the Ricci flow equation，characterized by a Riemannian metric g and a smooth potential function f that together satisfy the equation： $Ric+\nabla^2 f=\lambda g$

Where $\nabla^2 f$ is the Hess(f)，$\lambda$ is a real constant。

在研究 Ricci 流的奇異性（如形成奇點的過程）時扮演關鍵角色。

Key proterties：

1. Self-similarity：Under the Ricci flow ($\partial_t g=-2Ric$)，the metric evolves by diffeomorphisms and scaling。它們對應於 Ricci 流的自相似解，即度量只是在同構變換或縮放變換下演化，而保持相同的幾何特性。
2. Classifiation
3. Eintein metrics as trivial cases

Examples：

1. Gaussian soliton
2. Bryant soliton
3. Cigar soliton