§ 隨波逐流的S^3

For S^n of radius r(t)，the metric is given $g = r^2 \overline{g}$，where $\overline{g}$ is the metric on the unit sphere。The sectional curvature are all $\frac{1}{r^2}$。

The metric of unit 3-sphere，$\overline{g} = ds^2 = d\psi^2 + \sin^2\psi(d\theta^2 + \sin^2\theta d\phi^2)$

And of 3-sphere with radius r=r(t)，$g = ds^2 = r^2 d\psi^2 + r^2 \sin^2\psi(d\theta^2 + \sin^2\theta d\phi^2)$

$g = r^2 \overline{g}$

For a n-sphere，Ric(g)=(n-1)g，so the Ricci flow equation becomes a ODE。

$$\frac{\partial g}{\partial t} = -2Ric(g) \Rightarrow \frac{\partial}{\partial t}(r^2 \overline{g}) = -2(n-1)\overline{g} \Rightarrow \frac{dr^2}{dt} = -2(n-1)$$

$r^2 = R_0^{\ 2} - 2(n-1)t$

$r(t) = \sqrt{R_0^{\ 2} - 2(n-1)t}$，as $t \to \frac{R_0^{\ 2}}{2(n-1)}$，the sphere shrinks to a point(a singularity)。

Where n=3

Similarly，for hyperbolic n-space $H^n(n>1)$，the Ricci flow reduces to the ODE

$\frac{d(r^2)}{dt} = 2(n-1)$ which has the solution $r(t) = \sqrt{R_0^{\ 2} + 2(n-1)t}$

So the solution expands out to infinity。