Chaotic Dynamical Behaviour in Soliton Solutions for a New (2+1)Dimensional Long Dispersive Wave System

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Chaotic Dynamical Behaviour in Soliton Solutions for a New (2+1)-Dimensional Long Dispersive Wave System *

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With the help of variable separation approach, a quite general excitation of a new (2+1)-dimensional long dispersive wave system is derived. The chaotic behaviour, such as chaotic line soliton patterns, chaotic dromion patterns, chaotic–period patterns, and chaotic–chaotic patterns, in some new localized excitations are found by selecting appropriate functions.

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Chaos and solitons are two important phenomena of nonlinearity. They are widely applied in many natural sciences, particularly in the fluid dynamics, plasma physics, field theory, optics and condensed matter physics, etc. Usually, one considers that solitons are the basic excitations of the integrable models while chaos are the basic behaviour of the nonintegrable models. Thus, the soliton and chaotic behaviours are usually studied in two distinguished branches of theory. As is known, there is chaotic behaviour in some lower-dimensional physical models. Recently, Lou *et al.* have found chaotic behaviour in some higher-dimensional soliton systems by making use of lower dynamical chaos systems.[1] In this Letter, we study the further chaotic behaviour of soliton solution for a new (2+1)-dimensional long dispersive wave (NLDW) system

$$\lambda q_t + q_{xx} - 2q\int (qp)_x \mathrm{d}y = 0, \tag{1}$$

$$\lambda p_t - p_{xx} + 2p\int (qp)_x \mathrm{d}y = 0, \tag{2}$$

where $\partial_x = \partial_\xi$, $\partial_y = \partial_\xi - \lambda\partial_\eta$, and λ is a constant. This system apparently differs from the usual or traditional long dispersive wave system.[2] The real version of this system was obtained as a reduction of self-dual Yang–Mills field equation which was first introduced by Chakravarty *et al.*[3] while the complex version appears in Ref. [4]. The system has the Painlevé properties as was shown by Radha and Lakshmanan[5] (real version) and Porsezian[6] (complex version). The bilinear method was applied in Ref. [5] to obtain some soliton and dromion solutions. It is interesting to note that the system can be reduced to a single nonlocal equation introduced by Fokas in the form

$$\mathrm{i}\lambda q_t + q_{xx} - 2q\int |q|_x^2 dy = 0, \tag{3}$$

where p is complex conjugation of q, i.e., $p = q^*$ and $t \to \mathrm{i}t$.

In order to investigate Eqs. (1) and (2) conveniently, we first make use of the transformation $qp = w_y$, where w is some arbitrary potentials. Therefore, Eqs. (1) and (2) can be converted into a system of three partial differential equations

$$\lambda q_t + q_{xx} - 2qw_x = 0, \tag{4}$$

$$\lambda p_t - p_{xx} + 2pw_x = 0, \tag{5}$$

$$qp = w_y. \tag{6}$$

Searching for soliton solutions of a nonlinear model, one can use different kinds of methods. One of the powerful methods is the variable separation approach, which was recently presented by Lou[7] and successfully applied in some (2+1)-dimensional models.[8–15] Now we use this method to discuss the (2+1)-dimensional NLDW system. To solve the system (4)–(6), we first take the following Bäcklund transformation

$$q = \frac{Q}{f}, \quad p = \frac{P}{f}, \quad w = -(\ln f)_x + F_0(x,t), \tag{7}$$

which can be derived from the standard Painlevé truncated expansion, where f, P, and Q are arbitrary differential functions of the arguments $\{x, y, t\}$, and $F_0(x,t)$ is an arbitrary seed solution. Substituting Eq. (7) directly into Eqs. (4)–(6) yields the bilinear form

$$(D_x^2 + \lambda D_t)Q\cdot f - 2fQF_{0x} = 0, \tag{8}$$

$$(D_x^2 - \lambda D_t)P\cdot f - 2fPF_{0x} = 0, \tag{9}$$

$$D_xD_y f\cdot f + 2PQ = 0, \tag{10}$$

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where D_x, D_y, and D_t are the usual bilinear operators introduced first by Hirota.[16]

In order to find some interesting solutions of Eqs. (8)–(10), we further use the variable separation ansatz

$$f = a_1F + a_2G + a_3FG, \quad Q = F_1G_1\exp[\lambda(r+s)],$$
$$P = \frac{F_1G_1}{\exp[\lambda(r+s)]}, \tag{11}$$

where a_1, a_2, a_3 are arbitrary constants and $F \equiv F(x,t)$, $G \equiv G(y,t)$, $F_1 \equiv F_1(x,t)$, $G_1 \equiv G_1(y,t)$, $r \equiv r(x,t)$, $s \equiv s(y,t)$ all are arbitrary functions of the indicated variables.

Substituting the ansatz Eq. (11) into Eq. (10) yields

$$F_1^2G_1^2 - a_1a_2F_xG_y = 0. \tag{12}$$

Since the functions F and F_1 are only functions of $\{x,t\}$ and the functions G and G_1 are only functions of $\{y,t\}$, Eq. (12) can be solved by the following variable separated equation

$$F_1 = \delta_1\sqrt{a_1a_2c_0^{-1}F_x},\ G_1 = \delta_2\sqrt{c_0G_y},\ (\delta_1^2 = \delta_2^2 = 1), \tag{13}$$

where c_0 is an arbitrary function of time t.

Similar to the above procedure, substituting Eq. (11) together with Eq. (13) into Eqs. (8) and (9) yields the following variable separated equations

$$-a_1a_2F_t = 2a_1a_2r_xF_x + c_1F^2 + c_2a_2F + c_3a_2^2, \tag{14}$$

$$a_1^2G_t = (c_1 - c_2a_3 + c_3a_3^2)G^2 + a_1(-c_2 + 2c_3a_3)G + a_1^2c_3, \tag{15}$$

$$s = g(y) + b(t), \tag{16}$$

$$8F_x^2F_{0x} = 2F_xF_{xxx} + 4\lambda^2F_x^2(r_x^2 + r_t + b_t) - F_{xx}^2, \tag{17}$$

where $g(y)$ and $b(t)$ are arbitrary functions of indicated variables, and c_1, c_2, and c_3 are arbitrary functions of time t.

It is still difficult to obtain general solutions of Eqs. (14), (15) and (17) for any fixed $F_0(x,t)$. Fortunately, we can deal the problem with an alternative way. Since F_0 is an arbitrary seed solution, we can view F as an arbitrary function of $\{x,t\}$. The function r can be expressed by F simply by integration of Eq. (14). Then the seed solution F_0 can be fixed by Eq. (17). The result reads

$$r_x = \frac{-1}{2a_1a_2F_x}(c_1F^2 + c_2a_2F + c_3a_2^2 + a_1a_2F_t), \tag{18}$$

$$F_{0x} = \frac{1}{8F_x^2}[2F_xF_{xxx} + 4\lambda^2F_x^2(r_x^2 + r_t + b_t) - F_{xx}^2]. \tag{19}$$

As to the Riccati Eq. (15), its general solution has the form

$$G(y,t) = \frac{A_1(t)}{A_2(t) + U(y)} + A_3(t), \tag{20}$$

where $U(y)$ is an arbitrary function of y while A_1, A_2 and A_3 are arbitrary functions of time t, which are linked with c_1, c_2 and c_3 by

$$c_1 = \frac{-1}{A_1}(2a_1a_3A_3A_{2t} + a_1a_3A_{1t} + a_1^2A_{2t} - a_3^2A_1A_{3t} + a_3^2A_3^2A_{2t} + a_3^2A_3A_{1t}), \tag{21}$$

$$c_2 = \frac{-1}{A_1}(2a_1A_3A_{2t} + a_1A_{1t} - 2a_3A_1A_{3t} + 2a_3A_3^2A_{2t} + 2a_3A_3A_{1t}), \tag{22}$$

$$c_3 = \frac{-1}{A_1}(A_3A_{1t} + A_3^2A_{2t} - A_1A_{3t}). \tag{23}$$

Using Eq. (21)–(23), Eq. (15) becomes

$$G_t = \frac{-1}{A_1}[A_{2t}G^2 - (A_{1t} + 2A_3A_{2t})G + A_3^2A_{2t} + A_3A_{1t} - A_1A_{3t}]. \tag{24}$$

One can verify directly that Eq. (20) is a general excitation of Eq. (24).

Finally, substituting Eq. (11) together with Eqs. (13)–(17) into Eq. (7), we derive a quite general excitation of the (2+1)-dimensional system Eqs. (4)–(6),

$$q = \frac{\delta_1\delta_2\sqrt{a_1a_2F_xG_y}\exp[\lambda(r + g(y) + b(t))]}{a_1F + a_2G + a_3FG}, \tag{25}$$

$$p = \frac{\delta_1\delta_2\sqrt{a_1a_2F_xG_y}}{(a_1F + a_2G + a_3FG)\exp[\lambda(r + g(y) + b(t))]}, \tag{26}$$

$$w = F_0 - \frac{a_1F_x + a_3F_xG}{a_1F + a_2G + a_3FG}, \tag{27}$$

with four arbitrary functions $F(x,t)$, $G(y,t)$, $g(y)$ and $b(t)$, while r and F_0 are determined by Eqs. (18) and (19). In the following discussion, we study the structure of the potential qp. From Eqs (25) and (26), we have

$$qp = \frac{a_1a_2F_xG_y}{(a_1F + a_2G + a_3FG)^2}. \tag{28}$$

Because of the arbitrariness of the functions F and G included in Eq. (28), the quantity qp obviously possesses quite rich structures. Now the important question is whether we can find some new types of soliton structures which possess chaotic behaviour, i.e., the chaotic localized excitations for the (2+1)-dimensional NLDW system. The answer is apparently positive since $F(x,t)$ and $G(y,t)$ are arbitrary functions. There are various chaotic dromion and lump excitations because any types of (1+1)- and/or (0+1)-dimensional chaos and fractal models can be used to construct localized excitations of high-dimensional models. Some interesting possible chaotic patterns are cited here. For simplification in the following discussions, we set $a_1 = a_2 = 1$ and $a_3 = 2$ in the expression Eq. (28).

(1) Chaotic dromions patterns

In (2+1)-dimensions, one of the most important nonlinear solutions is the dromion excitation, which is localized in all directions. Now we set F and G as

$$F = 1 + \exp(x), \quad G = 1 + (100 + f(t))\exp(y), \tag{29}$$

where $f(t)$ is arbitrary function of time t. From the excitation Eq. (28) with Eq. (29), one knows that the amplitude of the dromion is determined by the function $f(t)$. If we select the function $f(t)$ as a solution of a chaotic system, then we can obtain a type of chaotic dromion solution. In Fig. 1(a), we exhibit the shape of the dromion for the physical quantity qp shown by Eq. (28) at a fix time (for $f(t) = 0$) with Eq. (29). The amplitude A of the dromion is changed chaotically with $f(t)$ as depicted in Fig. 1(b), where $f(t)$ is a chaotic solution of the following Rössler system[17]

$$f_t = -g - h, \quad g_t = f + ag, \quad h_t = b + h(f - c), \tag{30}$$

where $a = b = 0.2$, $c = 5.7$.

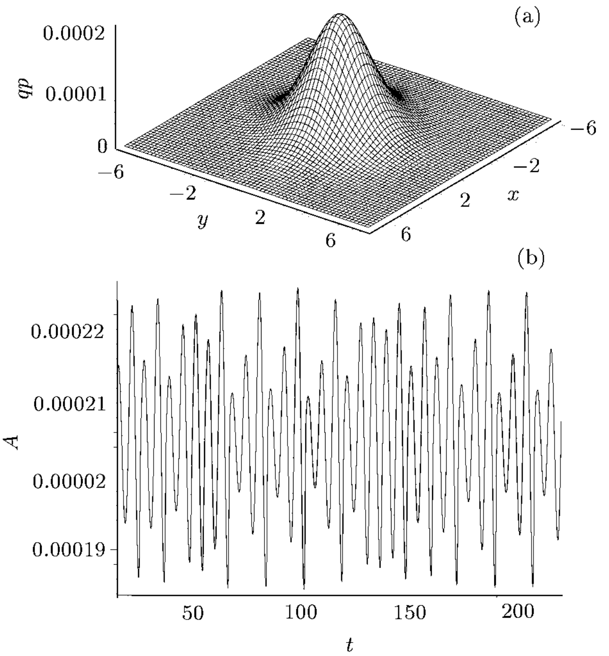

Fig. 1. (a) Single dromion structure for the physical quantity qp given by Eq. (28) with the conditions (29) and $f(t) = 0$. (b) Evolution of the amplitude A of chaotic dromion related to (a) with $f(t)$ being a chaos solution of the Rössler system Eq. (30) at different times.

(2) Chaotic line solitons patterns.

It is interesting that the localized excitations are not only chaotic with time t, but also with space, that is to say, in direction x or/and y. If one of F and G is chosen as a localized function while the other one is chaotic solutions of some (1+1)-dimensional (or (0+1)-dimensional) nonintegrable models, then the excitation Eq. (28) becomes a chaotic line soliton which may be chaotic in x or y direction. For example, we may set F or G as the solutions of the celebrated Rössler system ($\varsigma = x + \omega t, \eta = y + \nu t$)

$$F_\varsigma = -g - h, \quad g_\varsigma = F + ag, \quad h_\varsigma = b + h(F - c), \tag{31}$$

or

$$G_\eta = -g - h, \quad g_\eta = G + ag, \quad h_\eta = b + h(G - c). \tag{32}$$

Figure 2(a) shows the chaotic line soliton pattern expressed by Eq. (28) with the selections: F is a chaotic solution of Rössler system Eq. (31) and

$$G = 200 + \tanh(y). \tag{33}$$

while the parameters are selected to be

$$a = 0.2, \quad b = 0.2, \quad c = 5.7, \quad t = 0. \tag{34}$$

Fig. 2(b) is a typical plot of chaotic solution F in the Rössler system Eq. (31).

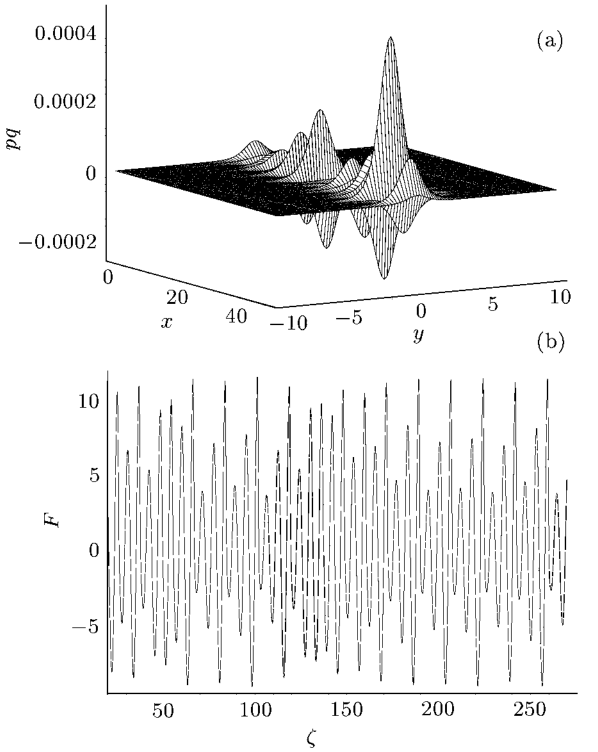

Fig. 2. (a). Chaotic line soliton structure for the physical quantity qp given by Eq. (28) with the conditions (31), (33) and (34) at $t = 0$. (b) Typical plot of chaotic solution F in the Rössler system Eq. (31).

(3) Chaotic-periodic patterns

Now we select one of F and G as a periodic function while the other one as chaotic, the solution expressed by Eq. (28) will become some chaotic–periodic patterns which are chaotic in one direction and periodic in other direction.

Figure 3(a) shows a special chaotic–periodic pattern of expression Eq. (28) under conditions G being 200 plus the chaotic solution of Eq. (32) with Eq. (34) while F being the periodic solution of Eq. (31) with

$$a = 0.2, \quad b = 0.2, \quad c = 3.5. \tag{35}$$

Fig. 3(b) is a plot of the typical two-periodic solution of Rössler system Eq. (31) with Eq. (35).

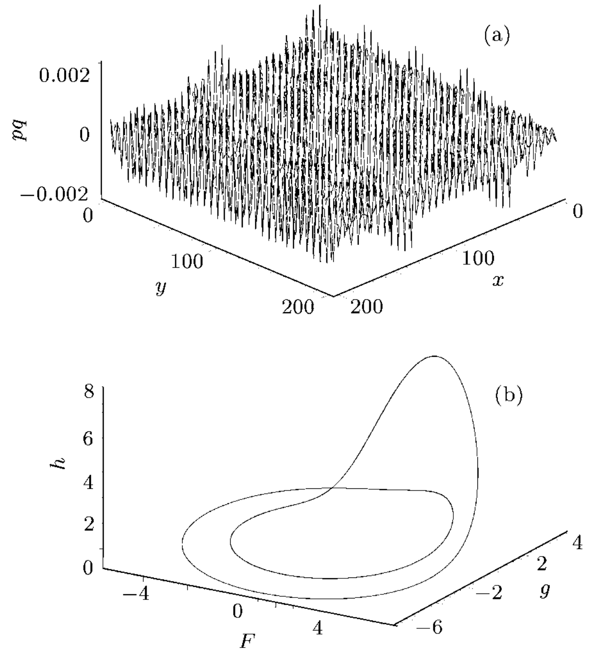

Fig. 3. (a). Chaotic-periodic pattern for the physical quantity qp given by Eq. (28) and the functions G and F being the typical chaotic solution and periodic solution of the Rössler system Eq. (32) with Eq. (34) and the system Eq. (31) with Eq. (35), respectively. (b) Typical two-periodic solution of the Rössler system Eq. (31) with Eq. (35).

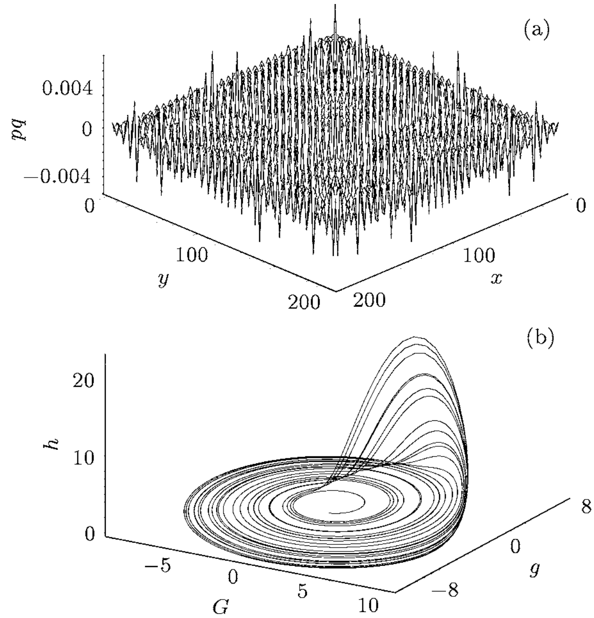

Fig. 4. (a) Chaotic-chaotic pattern for the physical quantity qp given by the expression Eq. (28) and the function F and G being the chaotic solution of the Rössler system Eq. (31) and system Eq. (32) with condition (34). (b) Typical chaotic solution of the Rössler system Eq. (32) with Eq. (34).

(4) Chaotic-chaotic patterns

Furthermore, if F and G both are chosen as chaotic solution of the (1+1)-dimensional (or (0+1)-dimensional) nonintegrable models, then the expression Eq. (28) becomes some types of (2+1)-dimensional space–time patterns which may be chaotic in the x and y directions. The related plots are depicted in Figs. 4(a) and 4(b).

In summary, by means of the variable separation approach, the (2+1)-dimensional NLWD system has successfully been solved. By selecting the arbitrary functions appropriately, we find some new localized excitations which possesses the chaotic dynamical behaviour. Why can a dromion and/or a soliton be chaotic? In Ref. [1], the authors have made a brief analysis. Another problem baffles us. Generally, chaos are the opposite circumstances to solitons in nonlinear science because solitons are the representatives of integrable system while chaos represent the behalf of non-integrable systems. However, in this work, we find some chaotic dynamical behaviour for dromion or soliton solutions for the (2+1)-dimensional NLWD model. Naturally, just as is pointed in Ref. [18], the question of what is really the integrability definition casts on researcher's mind, so does the question of how to find and make use of this novel phenomena in reality.

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