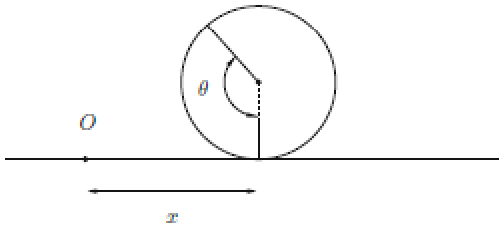
例 1. Wheel rolling without slipping



$\dot{x} = R\dot{\theta}$

Configuration space is $R \times S^1$

$X = R\frac{\partial}{\partial x} + \frac{\partial}{\partial \theta}$, the kernel of the 1-form $\omega = dx - Rd\theta$

$d\omega = 0$ this is a semi-holonomic constraint , corresponding to an integrable distribution.

The leaves of the distribution are the submanifolds with equation $x = x_0 + \mathrm{R}\theta$

Since $\mu^{-1}\mathcal{R}$ is orthogonal to the constraint for a perfect reaction force $\mathcal{R}$, the constraint must be in the kernel of $\mathcal{R}$, and hence $\mathcal{R} = \lambda\omega$ for some smooth function $\lambda : TM \to \mathbb{R}$.

If the kinetic energy of the wheel is

$$K = \frac{M}{2}(v^x)^2 + \frac{I}{2}(v^\theta)^2$$

then

$$\mu\left(\frac{D\dot{c}}{dt}\right) = M\ddot{x}dx + I\ddot{\theta}d\theta.$$