§ Mechanical Systems

Definition

A mechanical system is $(M,<.,.>,F)$

1. M is a differentiable manifold，called the configuration space。(位相空間)
2. $<.,.>$ is a Riemannian metric on M yielding the mass operator
   $\mu: TM \to T^*M$ by $\mu(v)(\omega) = <v,\omega>$...(1)
3. $F: TM \to T^*M$ called external force
4. A motion of the mechanical system is a solution $c: I \to M$ of the Newton

   equation $\mu(\frac{D\dot{c}}{dt}) = F(\dot{c})$...(2)

   $\frac{D\dot{c}}{dt}$是加速度 a。F=ma

The geodesics of a Riemannian manifold are the motions for F=0

Definition

若存在$U: M \to R$ 使得$F(v) = -(dU)_{\pi(v)}$ ...(3)則稱 F 為守恆(conservative)。

U 稱為位能(potential energy)。

動能(kinetic energy) $K: TM \to R$ $K(v) = \frac{1}{2} <v,v>$...(4) 。($K = \frac{1}{2}mv^2$)

定理

能量$E(t) = K((\dot{c}(t)) + U(c(t))$ ，在一守恆系中，E(t)是一常數

E(t) is a constant along any motion c

Proof

$$\frac{dE(t)}{dt} = \frac{d}{dt}(\frac{1}{2} <\dot{c}(t),\dot{c}(t)> + U(c(t)) = <\frac{D\dot{c}}{dt}(t),\dot{c}(t)> + (dU)_{c(t)}\dot{c}(t)$$

$$= \mu(\frac{D\dot{c}}{dt})(\dot{c}) - F(\dot{c})(\dot{c}) = 0$$

把(1)~(4)兜起來。