第四章 習作

1. The Levi-Civita connection on a **torus** $T^n = \mathbb{R}^n / \Gamma$ is induced by the Euclidean connection on $\mathbb{R}^n$. In particular, each torus carries a flat metric.

2. Compute the Christoffel symbols of the Levi-Civita connection on the **sphere** S^n and on **hyperbolic space** H^n in the coordinates given by stereographic projection, that is, (1.1.2) and (1.1.9).

3. Compute the curvature tensor of the sphere $S^n_r := \{x \in \mathbb{R}^{n+1} : |x| = r\}$ of radius r.

1. Compute the transformation behavior of the Christoffel symbols of a connection under coordinate transformations.

2. Let E be a vector bundle with fiber $\mathbb{C}^n$ and a Hermitian bundle metric. Develop a theory of unitary connections, i.e. of connections respecting the bundle metric.

3. Show that each vector bundle with a bundle metric admits a metric connection.

4. Let $x_0 \in M$, D a flat metric connection on a vector bundle E over M. Show that D induces a map $\pi_1(M, x_0) \to \mathrm{O}(n)$, considering $\mathrm{O}(n)$ as the isometry group of the fiber E_{x_0}.

5. Verify the formula $\not{\partial} = cl \circ \nabla$ given in §4.4.