§ Spin bundle S_n

Lemma 4.4.3

For smooth sections μ of $Cl^C(M)$，σ of S_n

$\nabla(\mu\sigma) = \nabla(\mu)\sigma + \mu\nabla(\sigma)$

Where the products are Clifford multplication。

自旋叢是一個向量叢，它的每個截面（section）被稱為一個旋量場（spinor field）。

The Dirac operator D 作用在 Spin bundle S_n 的截面 σ 上：

$D\sigma(x) = e_i\nabla_{e_i}(\sigma)(x)$ 其中 $\{e_i\}$ 是 T_xM 的么正基。右邊的乘積是 Clifford 乘積。

Example We consider the case of $\mathbb{R}^2$ with coordinates x, y. Recalling the discussion in §2.6, the spinor space then is $\mathbb{C}^2$, and the vectors e_1 and e_2 act on spinors via

$$\gamma(e_1) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \gamma(e_2) = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}.$$

Writing a spinor field $\sigma : \mathbb{R}^2 \to \mathbb{C}^2$ in components as $\begin{pmatrix} \sigma^1 \\ \sigma^2 \end{pmatrix}$, we then have

$$\partial\!\!\!/\sigma = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \frac{\partial\sigma^1}{\partial x} \\ \frac{\partial\sigma^2}{\partial x} \end{pmatrix} + \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} \begin{pmatrix} \frac{\partial\sigma^1}{\partial y} \\ \frac{\partial\sigma^2}{\partial y} \end{pmatrix} = 2 \begin{pmatrix} \frac{\partial\sigma^2}{\partial \bar{z}} \\ -\frac{\partial\sigma^1}{\partial z}. \end{pmatrix} \tag{4.4.8}$$

Thus, in this case, the Dirac operator is simply the Cauchy–Riemann operator.