p.53 定義 2.1.7 後的一個習作

S^1 is a submanifold of R^2 ，TR^2 is the tangent bundle of R^2

If we restrict TR^2 to S^1 ，the tangent bundle has fiber $T_xS^1 = \{y \in R^2 \mid x \bullet y = 0\} \subset R^2$

Write down the explicit bundle charts of TS^1

1. 選取 S^1 的覆蓋(cover)
2. 設 diffeomorphism(微分同胚映射)

$U_1 = S^1 / \{(-1,0)\}$ ，$\theta : U_1 \to (-\pi, \pi)$ coordinates on U_1 ，$p(\cos\theta, \sin\theta) \in S^1$

A tangent vector $v \in T_pS^1$ at $p(\cos\theta, \sin\theta)$ is $v = c(-\sin\theta, \cos\theta)$

The bundle chart $\phi_1 : \pi^{-1}(U_1) \to U_1 \times R$ is defined by：

$\phi_1(p, v) = (\theta, c)$ where $v = c(-\sin\theta, \cos\theta)$

Under this chart，$\pi^{-1}(U_1)$ is idetified with $(-\pi, \pi) \times R$

$U_2 = S^1 / \{(1,0)\}$ ，$\varphi : U_2 \to (0, 2\pi)$ coordinates on U_2 ，$p(\cos\varphi, \sin\varphi)$

A tangent vector $v \in T_pS^1$ at $p(\cos\varphi, \sin\varphi)$ is $v = d(-\sin\varphi, \cos\varphi)$

The bundle chart $\phi_2 : \pi^{-1}(U_2) \to U_2 \times R$ is defined by：

$\phi_2(p, v) = (\varphi, d)$ where $v = d(-\sin\varphi, \cos\varphi)$

Under this chart，$\pi^{-1}(U_2)$ is idetified with $(0, 2\pi) \times R$

On $U_1 \cap U_2 = S^1 / \{(-1,0),(1,0)\}$

If $\theta \in (0, \pi)$ then $\varphi = \theta$

If $\theta \in (-\pi, 0)$ then $\varphi = 2\pi + \theta$

For a tangent vector v，we have c=d in both cases，so the transition functions (轉換函數 define how coordinates change between overlapping charts 重疊區) is $(\theta, c) \to (\varphi, c)$ with the identity on the fiber component。

$p \in U_1 \cap U_2$ 存在連續可逆函數 $\phi_1 \circ \phi_2^{-1} : (U_1 \cap U_2) \times R \to (U_1 \cap U_2) \times R$ 用 g_{12} 表示，稱為轉換函數，是一個矩陣。

G=$\{g_{\alpha\beta}\}$ 稱為 E 的結構群 $\subset GL(n, R)$

滿足

$g_{\alpha\alpha}(p)=1$

$g_{\alpha\beta}(p)=g_{\beta\alpha}(p)^{-1}$

$g_{\alpha\beta}(p)\circ g_{\beta\gamma}(p)g_{\gamma\alpha}(p)=1$ for $p\in U_\alpha\cap U_\beta\cap U_\gamma$

Thus，the transition function $g_{12}:U_1\cap U_2\to GL(1,R)=R/\{0\}$ is constant and equal to 1。

註：

一個 bundle chart(叢圖表)包含：

(1) 一個底空間的開集 $U\subseteq M$ (2) 一個 diffeomorphism ϕ