§ symplectic structure of TM

$j_\theta : T_\theta TM \to T_x M \times T_x M$

$j_\theta(\xi) = (d_\theta \pi(\xi), K_\theta(\xi))$

If $\xi = (\xi_h, \xi_v) \leftrightarrow j_\theta(\xi)$ is an identification，that is $\xi_h = d_\theta \pi(\xi), \xi_v = K_\theta(\xi)$

Sasaki metric

$$<<\xi,\eta>>_\theta := <d_\theta \pi(\xi), d_\theta \pi(\eta)>_{\pi(\theta)} + <K_\theta(\xi), K_\theta(\eta)>_{\pi(\theta)}$$

$J_\theta : T_\theta TM \to T_\theta TM$

$J_\theta(\xi_h, \xi_v) = (-\xi_v, \xi_h)$ a almost complex structure

Define a smyplectic form $\Omega_\theta(\xi,\eta) := << J_\theta \xi, \eta >>_\theta$

Exercise

Show that

1. Ω_θ is antisymmetric and nondegenerate
2. For each θ，J_θ is a linear isometry of the Sasaki metric
3. J_θ is skew symmetric relative to the Sasaki metric

$H(x,v) := \frac{1}{2} <v,v>_x$ is the energy function

The geodesic vector field is given by $G : TM \to TTM$

$G(\theta) := \frac{\partial}{\partial t}|_{t=0}\, \phi_t(\theta) = \frac{\partial}{\partial t}|_{t=0}\, (\gamma_\theta(t), \dot{\gamma}_\theta(t))$ Where γ_θ is the unique geodesic with initial condition $\theta = (x,v)$

命題

$dH = \iota_G \Omega$ or equivalently $d_\theta H(\xi) = \Omega_\theta(G(\theta), \xi)$ for all $\theta \in TM$ ，$\xi \in T_\theta TM$

證明