§ 6.4 General Relativity

Gravity can be introduced in Newtonian mechanics through the symmetric Cartan connections，which preserves Galileo spacetime structure。

A natural idea for introducing gravity in special relativity is then to search for symmetric connections preserving the Minkowski inner product。

The Minkowski spacetime is a Lorentzian manifold。

If $c: I \to M$ is a geodesic，then $<\dot{c},\dot{c}>$ is constant，as

$$\frac{d}{ds}<\dot{c}(s),\dot{c}(s)>=2<\frac{D\dot{c}(s)}{ds},\dot{c}(s)>=0$$

A geodesic is called

(1)timelike $<\dot{c},\dot{c}><0$ We take timelike geodesic to represent the free-falling motions of massive particles。This ensures that the equivalence principle holds。

(2)null $<\dot{c},\dot{c}>=0$ Null geodesics will be taken to represent the motions of light rays。

(3)spacelike $<\dot{c},\dot{c}>>0$

In timelike case，the proper time measured by the particles between events c(a) and c(b) is $\tau(c)=\int_a^b \left|\dot{c}(s)\right| ds$ ，where $|v|=|<v,v>|^{\frac{1}{2}}$ for any $v \in TM$

Definition

The Lorentzian manifold (M ,g) is said to be a vacuum solution of the Einstein field equation if its Levi-Civita connection satisfies Ric=0

General Einstein field equation

$$Ric-\frac{S}{2}g=8\pi E$$

Where $S=\sum_{\mu,\nu=0}^{3} g^{\mu\nu}R_{\mu\nu}$ is the scalar curvature and E is the so-called energy-momentum tensor of the matter content of the spacetime。