§ 6.1 The Cartan Connection

Let $(x^0, x^1, x^2, x^3) = (t, x, y, z)$ be an inertial frame on Galileo spacetime。

ϕ gravitational potential，ρ matter density

$\frac{d^2 x^i}{dt^2} = -\frac{\partial \phi}{\partial x^i}$ i=1，2，3

$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 4\pi\rho$ …Poisson equation

$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$ …Laplace equation as in vacuum

Cartan connection

$\Gamma^i_{00} = \frac{\partial \phi}{\partial x^i}$，i=1，2，3，geodesic equation $\ddot{x}^i + \Gamma^i_{jk} \dot{x}^j \dot{x}^k = 0$

1. $\omega^i_0 = \frac{\partial \phi}{\partial x^i} dt$

2. $\Omega^\mu_\nu = d\omega^\mu_\nu + \sum_{\alpha=0}^{3} \omega^\mu_\alpha \wedge \omega^\alpha_\nu$

$$\Omega^i_0 = \sum_{j=1}^{3} \frac{\partial^2 \phi}{\partial x^j \partial x^i} dx^j \wedge dx^i$$

$$Ric = \left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2}\right) dt^2$$