§ Minimal Coupling Principle 最小耦合原理

1. Take a law of physics valid in inertial coordinates in flat spacetime。
2. Write it in a coordinate-invariant form(tensorial form)。
3. Assert that the resulting law remains true in curved spacetime。

例

The motion of freely-falling particle，the parametrized curve $x^{\mu}(\lambda)$

$\frac{d^2x^{\mu}}{dx^2}=0$ ($\frac{dx^{\mu}}{dx}$ is s tensorial form，but $\frac{d^2x^{\mu}}{dx^2}$ is not)

By chain rule $\partial_{\nu}(\frac{dx^{\mu}}{d\lambda})=\frac{d^2x^{\mu}}{d\lambda^2}\times\frac{d\lambda}{dx^{\nu}}$

$\frac{d^2x^{\mu}}{dx^2}=\frac{dx^{\nu}}{d\lambda}(\partial_{\nu}\frac{dx^{\mu}}{d\lambda})$ ，把∂_{ν}改寫成 covariant derivative ∇_{ν}

Note that $\nabla_{\mu}V^{\nu}=\partial_{\mu}V^{\nu}+\Gamma^{\nu}_{\mu\rho}V^{\rho}$

$\nabla_{\nu}\frac{dx^{\mu}}{d\lambda}=\partial_{\nu}\frac{dx^{\mu}}{d\lambda}+\Gamma^{\mu}_{\nu\sigma}\frac{dx^{\sigma}}{d\lambda}=\frac{d^2x^{\mu}}{d\lambda^2}\times\frac{d\lambda}{dx^{\nu}}+\Gamma^{\mu}_{\nu\sigma}\frac{dx^{\sigma}}{d\lambda}$ 所以

$\frac{d^2x^{\mu}}{dx^2}=\frac{dx^{\nu}}{d\lambda}(\nabla_{\nu}\frac{dx^{\mu}}{d\lambda})=\frac{d^2x^{\mu}}{d\lambda^2}+\Gamma^{\mu}_{\nu\sigma}\frac{dx^{\nu}}{d\lambda}\frac{dx^{\sigma}}{d\lambda}=0$ ， 把ν改成ρ

$\frac{d^2x^{\mu}}{d\lambda^2}+\Gamma^{\mu}_{\rho\sigma}\frac{dx^{\rho}}{d\lambda}\frac{dx^{\sigma}}{d\lambda}=0$ 即為 geodesic equation