§ 3.17 The Schwarzschild Metric

S.Chandrasekhar 1910-1995 1983 諾貝爾物理獎(星體結構)
John Linghton Synge (辛格) 1897-1995 Relativity：the general theory
Karl Schwarzschild(史瓦西) 1873-1916

Chandra 錢德拉 梵文是 “月亮””發光的”意思 。
錢德拉的”白矮星理論”備受愛丁頓攻擊，直到 1983 年以星體結構理論得到諾貝爾物理獎。
史瓦西在 1915 年愛因斯坦發表廣義相對論不久即得到”黑洞”解。
在黎曼幾何有辛格定理：
M 是一正截曲率的閉黎曼流形 則
1. 若 M 是偶維數且可定向，則 M 是單連通。
2. 若 M 是奇維數，則它可定向。
單連通(simply connected)的意思是 M 上的任意封閉曲線必能在 M 上變形而縮為一點。例如 球面是單連通，環面則否。 [大域微分幾何 p.196]
錢德拉與辛格都有許多著作，應是都值得一讀。

The Schwarzschild metric is a spherically symmetric solution of Einstein's equation for the vacuum。
Follow Synge，we define a spherically symmetric space-time as a manifold $S^2 \times U^2$ ，where S^2 is a unit two-dimensional sphere，we take the usual polar coordinates (θ,φ) ，with the metric $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$
And a two-dimensional manifold U^2 with indefinite metric。

On U_2, since it is characterized by an indefinite metric, null lines

$$u = \text{constant} \qquad \text{and} \qquad v = \text{constant} \tag{2}$$

must exist. We shall take them as a basis for a coordinate system on U_2. With this choice of coordinates on S_2 and U_2, we can write the most general metric for a spherically symmetric space-time in the form

$$ds^2 = 4f\,du\,dv - e^{2\mu_3}[(d\theta)^2 + (d\varphi)^2 \sin^2\theta], \tag{3}$$

where f and μ_3 are functions of u and v.

$$ds^2 = -(1-\frac{2GM}{r})dt^2 + (1-\frac{2GM}{r})^{-1}dr^2 + r^2 d\Omega^2$$

張海潮先生的文章中寫成：

$$c^2 d\tau^2 = c^2(1-\frac{2GM}{rc^2})dt^2 - (1-\frac{2GM}{rc^2})^{-1}dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2$$

其中 M 是太陽的質量，c 是慣性座標下真空中的光速。

In order that we may transcribe the formulae of Chapter 2 directly, it is convenient to write

$$f = e^{2\mu_2},\ e^{2\psi} = e^{2\mu_3}\sin^2\theta,\ \mathrm{d}x^1 = \mathrm{d}\varphi,\ \mathrm{d}x^3 = \mathrm{d}\theta,\quad \text{and} \quad \mathrm{d}x^4 = i\,\mathrm{d}x^0, \quad (6)$$

when the metric becomes

$$-\mathrm{d}s^2 = e^{2\mu_2}(\mathrm{d}x^4)^2 + e^{2\psi}(\mathrm{d}x^1)^2 + e^{2\mu_2}(\mathrm{d}x^2)^2 + e^{2\mu_3}(\mathrm{d}x^3)^2. \quad (7)$$

以下要回到 CH02 計算 Riemann tensor and Ricci tensor

最後得到 Schwarzschild 當初得到的形式。

$$ds^2 = -(1-\frac{2GM}{r})dt^2 + (1-\frac{2GM}{r})^{-1}dr^2 + r^2 d\Omega^2$$

參考：

1. Schwarzschild radius https://highscope.ch.ntu.edu.tw/wordpress/?p=20477
2. RG3104