§ Smooth submanifolds

A manifold P is a submanifold of a manifold M provided：

1. P is a subset of M，and the inclusion map $j: P \to M$ is smooth
2. The differential maps of j are injective (that is，$(dj)(v) = 0 \Rightarrow v = 0$)
3. P is a topological subspace of M (that is，fas the relative topology)

For a submanifold P，it is customary to ignore dj and consider $T_p(P)$ to be a vector subspace of $T_p(M)$ 。

If P has dimension one less n=dim M it is called a hypersurface of M 。

Spherical coordinates

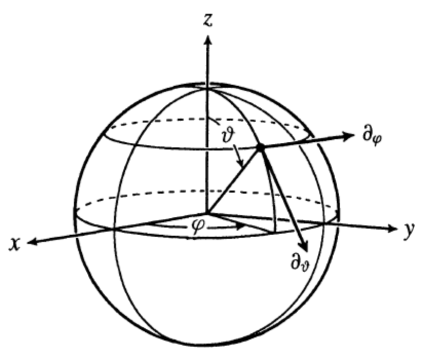

FIGURE 1.11. Spherical coordinates ϑ, φ on the unit 2-sphere.

Let S^2 be the sphere of unit radius in R^3 。Then spherical coordinates of R^3 retricted to S^2 give coordinates θ, φ on S^2 。

§ Foliations 葉理 岩石中的纖維狀組織

A distribution(平面場) Π of dimension k on a smooth manifold M is a smooth field of tangent k-planes on M。

A submanifold P of M such that $T_p(P) = \Pi_p$ for all $p \in P$ is called an integral manifold of Π 。If throught every point of M there is a integral manifold，then Π is integrable。

We say that a vector field V is in a distribution Π if $V_p \in \Pi_p$ for all $p \in M$

§ Frobenius Theorem

A distribution Π on M is integrable if and only if whenever vector fields X，Y are in Π ，their bracket [X,Y] is also in Π 。

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