§ $\frac{\partial}{\partial t}x(p,t)=\overrightarrow{H}(p,t)$

A geometric diffusion equation $\frac{\partial x}{\partial t}=\Delta_{M_t}x$ for the coordinates x of the corresponding family of surfaces $\{M_t\}_{t\in[0,T)}$ Where ∇_M is the Laplace-Beltrami operator。

Since $\Delta_{M_t}x=\overrightarrow{H}$ where $\overrightarrow{H}$ represents the mean curvature vector，we have

$$\frac{\partial}{\partial t}x(p,t)=\overrightarrow{H}(p,t)$$

MCF is the negative gradient flow for area。
It is a nonlinear PDE for the evolving the hypersurface that is similar to the ordinary heat equation。Model things such as cell，grain，and bubble growth。

Translating solution known as the Grim Reaper。

Suppose that M is a closed hypersurface in R^{n+1} and M_t is a variation of M。That is，M_t is a one-parameter family of hypersurface with $M_0=M$ 。
If we think of **volume** as a function o the space of hypersurfaces，then the first variation formula gives the derivative of volume under the variation

$$\frac{d}{dt}Vol(M_t)=\int_{M_t}<\partial_t x,Hn>$$

Here x is the position vector，n the unit normal，and H themean curvature scalar given by $H=div_{\mathrm{M}}(\mathrm{n})=\sum_{i=1}^{n}<\nabla_{e_i}n,e_i>$ where e_i is an orthoormal frame for M。
Equivalently，H is the sum of the principal curvatures of H。With this normalization，H is n/R on the round n-sphere of radius R。
It follows from the first variation formula that the gradient of volume is
$\nabla Vol=Hn$ and the most efficient way to reduce the volume is to choose the variation so that $\frac{\partial}{\partial t}x=-\nabla Vol=-Hn$
This negative gradient flow for volume is called MCF。

Under the MCF，a hypersurface locally moves in the direction where the volume element decreases the fastest。

thus，if M_t flows by MCF，then $\frac{d}{dt}Vol(M_t)=-\langle\nabla Vol,\nabla Vol\rangle=-\int_{M_t}H^2$ The flow constracts a closed hypersurface，eventually leading to its extinction in finite time。

Theorem

Given a compact，immersed hypersurface M in $\boldsymbol{R}^{n+1}$，there exists a unique mean curvature flow defined on the interval [0,T] with initial surface M。

Any closed smooth 4-dimensional manifold homotopy equivalent to S^4 can be smoothly emedded as a hypersurface。