

Solve the equation $u_x^2 + u_y^2 = e^{2y}$, $u(0,y)=0$

Assume a solution of the form $u(x, y) = e^y v(x)$

$$u_x = e^y v'(x) \quad , \quad u_y = e^y v(x)$$

$v'^2 + v^2 = 1$ This ODE has solutions $v(x)=\sin(x+c)$ or $v(x)=\cos(x+c)$

$$u(x, y) = e^y \sin x \quad \cdots \text{ANS}$$