Solve the equation

$u_{tt} - u_{xx} = 3e^x$ for $-\infty < x < \infty, -\infty < t < \infty$

$u(x,0) = 0, u_t(x,0) = 0$

Homogeneous solution $u_h(x,t) = f(x+t) + g(x-t)$

Let particular solution $u_p(x,t) = Ae^x \Rightarrow u_p(x,t) = -3e^x$

The general solution is $u(x,t) = f(x+t) + g(x-t) - 3e^x$

1. $u(x,0) = 0 \Rightarrow f(x) + g(x) = 3e^x$
2. $u_t(x,0) = 0$

   $u_t(x,t) = f'(x+t) - g'(x-t) \Rightarrow f'(x) = g'(x)$

   f(x)=g(x)++c

   we have $f(x) = \frac{1}{2}(3e^x + c), g(x) = \frac{1}{2}(3e^x - c)$

Since the problem does not provide additional boundary conditions，we can assume c=0 for simplicity。Thus，$f(x) = \frac{3}{2}e^x, g(x) = \frac{3}{2}e^x$

The general solution is：

$$u(x,t) = f(x+t) + g(x-t) - 3e^x = \frac{3e^x}{2}(e^t + e^{-t}) - 3e^x = 3e^x(\cosh t - 1)$$