§ Wave equation $u_{tt} - c^2 \Delta u = 0$

$$E(t) = \int_{-\infty}^{\infty} (\frac{1}{2} u_t^2 + \frac{1}{2} c^2 u_x^2) dx$$

$$\frac{dE}{dt} = \int_{-\infty}^{\infty} (u_t u_{tt} + c^2 u_x u_{xt}) dt = c^2 \int_{-\infty}^{\infty} (u_t u_{xx} + u_x u_{xt}) dt$$

$\int_{-\infty}^{\infty} u_t u_{xx} dx = u_t u_x \Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} u_{tx} u_x dx = -\int_{-\infty}^{\infty} u_x u_{xt} dx$ integrate $u_t u_{xx}$ by parts and

$u_x, u_t \to 0$ as $x \to \pm\infty$

Thus， $\frac{dE}{dt} = 0$ ，the total energy is constant over time。