

$$\S \quad au_{xx} + bu_{xy} + cu_{yy} + du_x + eu_y + fu = 0, \quad D = b^2 - 4ac$$

Let  $u = Ae^{\alpha x + \beta y}$  代入原方程

1. Transport equation  $u_t + cu_x = 0$

2. Wave equation  $c^2 u_{xx} - u_{yy} = 0, c > 0$   $D > 0$  , hyperbolic type

3. Heat equation  $u_t - ku_{xx} = 0$

Diffusion equation  $ku_{xx} - u_y = 0, k > 0$   $D = 0$  , parabolic type

4. Laplace equation  $u_{xx} + u_{yy} = 0$   $D < 0$  , elliptic type

5. Minimal surface  $\frac{\partial}{\partial x} \left( \frac{u_x}{\sqrt{1+u_x^2+u_y^2}} \right) + \frac{\partial}{\partial y} \left( \frac{u_y}{\sqrt{1+u_x^2+u_y^2}} \right) = 0$

$\S$

1. Linear homogeneous  $a(x, y)u_x + b(x, y)u_y + c(x, y)u = 0$

2. Linear inhomogeneous  $a(x, y)u_x + b(x, y)u_y + c(x, y)u = f(x, y)$