Consider the equation

$$\begin{cases} u_{xy} - \alpha u_{xx} = 0, (x,y) \in R^2 \\ u(0,y) = y, u_x(0,y) = 1 - y, y \in R \end{cases}$$

Solve the equation with $\alpha = 0, \alpha = 1$ respectively。

$$\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y} - \alpha \frac{\partial u}{\partial x}\right) = 0$$

$$\frac{\partial u}{\partial y} - \alpha \frac{\partial u}{\partial x} = f(y)$$

$$u = u(x,y) \quad \frac{du}{ds} = \frac{\partial u}{\partial x}\frac{dx}{ds} + \frac{\partial u}{\partial y}\frac{dy}{ds}$$

$$\frac{dx}{ds} = -\alpha, \frac{dy}{ds} = 1, \frac{du}{ds} = f(y)$$

$x + \alpha y = c$ ，y=s

Let $\xi = y, \eta = x + \alpha y$ 原方程變成$u_{\xi\eta} = 0$

$$u_{\xi} = \varphi(\xi)$$

$$u(x,y) = \int \varphi(\xi) d\xi + f(\eta) = f(x + ay) + g(y)$$

$\alpha = 0$ 時無解

$\alpha = 1$ 時$u(x,y) = x + y - xy - \frac{1}{2}x^2$