Solve the initial value problem

$$u_t + \frac{1}{2}(u^2)_x = 0, t > 0$$

($\frac{1}{2}(u^2)_x = uu_x$，所以是 Burger equation with shock formation。)

$$u(x,0) = \begin{cases} 1, x \le 0 \\ 1-x, 0 \le x \le 1 \\ 0, x \ge 1 \end{cases}$$

For $0 \le t < 1$:

- Region $x \le t$: Characteristics from $x_0 \le 0$ propagate with $u = 1$.
- Region $t < x < 1$: Characteristics from $0 \le x_0 \le 1$ yield $u = \frac{1-x}{1-t}$.
- Region $x \ge 1$: Characteristics from $x_0 \ge 1$ remain stationary with $u = 0$.

For $t \ge 1$:

- A shock forms at $t = 1$, starting at $x = 1$. The shock propagates with speed $\frac{1}{2}$ (from Rankine-Hugoniot condition).

Final Answer:

$$u(x,t) = \begin{cases} 1, & x \le \min(t, 1 + \frac{1}{2}(t-1)) \\ \frac{1-x}{1-t}, & t < x < 1 \text{ and } t < 1 \\ 0, & x \ge \max(1, 1 + \frac{1}{2}(t-1)) \end{cases}$$

More explicitly：

- For $t < 1$:

$$u(x,t) = \begin{cases} 1, & x \le t \\ \frac{1-x}{1-t}, & t < x < 1 \\ 0, & x \ge 1 \end{cases}$$

- For $t \ge 1$:

$$u(x,t) = \begin{cases} 1, & x \le 1 + \frac{1}{2}(t-1) \\ 0, & x > 1 + \frac{1}{2}(t-1) \end{cases}$$

Shock location for $t \ge 1$:

$x_s(t) = 1 + \frac{1}{2}(t-1).$