

台大 101 碩士班考題

1. Solve the ODEs and find its maximal interval of existence
 - (a) $y' = 1 + y^2, y(0) = y_0$
 - (b) $y' = y \ln y, y(0) = y_0 > 0$
2. (a) Find the general solutions of the ODE : $y' + \frac{1}{t}y = t^2$
 (b) Find the Laplace transform of the function $\cos \omega t$
3. Find the general solutions of the ODE : $x^2 y'' + bxy' + cy = 0$, where b, c are constants
4. Consider the logistic model $y' = ry(1 - \frac{y}{K}), y(0) = y_0$, where $r > 0, K > 0$ are two constants and $0 < y_0 < K$
 - (a) Find its general solutions
 - (b) Discuss the solution behavior (stable, unstable) as t tends to infinity
 - (c) Consider the harvest model $y' = ry(1 - \frac{y}{K}) - ey, 0 < y(0) = y_0 < K$, where $e > 0$ is the harvest rate . Discuss how the asymptotic solution depends on the harvest rate e
5. Consider the damped oscillation system with periodic forcing :
 $y'' + \alpha y' + \beta y = F_0 \cos(\Omega t)$ where $\alpha > 0, \beta > 0, F_0, \Omega$ are constants
 - (a) Find the solution to this system with initial condition $y(0) = y_0, y'(0) = v_0$
 - (b) Discuss the asymptotic behavior of the solutions (that is , what is the limit of $y(t)$ as $t \rightarrow \infty$)
6. Consider the conservative mechanical system in \mathbb{R} with a unit mass : $\ddot{x} = -V'(x)$ where $V(x)$ is the potential function and $-V'(x)$ is the force
 - (a) Show the energy $E(t) := \frac{1}{2} \left| \dot{x}(t) \right|^2 + V(x(t))$ is independent of time
 - (b) Show that if $V(x) \rightarrow \infty$ as $|x| \rightarrow \infty$ then all solutions with finite energy are periodic . (Constant solution is treated as a periodic solution .)
 - (c) Suppose the system has a damping term : $\ddot{x} = -V'(x) - \beta \dot{x}$, where $\beta > 0$ is the damping coefficient . Assume $V(x)$ is strictly convex and $V(x) \rightarrow \infty$ as $|x| \rightarrow \infty$. Show that all solutions tend to x_0 , the unique global minimum of V .