§ A conservative field

$F=(Ax\sin(\pi y),x^2\cos(\pi y)+Bye^{-z},y^2e^{-z})$ is conservative (curl F=0) A= B=

$$curlF=\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ Ax\sin(\pi y) & x^2\cos(\pi y)+Bye^{-z} & y^2e^{-z} \end{vmatrix}$$

Or differential form $\omega=Ax\sin(\pi y)dx+(x^2\cos(\pi y)+Bye^{-z})dy+y^2e^{-z}dz$ then

$d\omega=0$ implies $A=\frac{2}{\pi},B=-2$

此時存在φ (the potential) 使得$F=\nabla\varphi$

$\frac{\partial\varphi}{\partial x}=\frac{2}{\pi}\sin(\pi y)$ $\frac{\partial\varphi}{\partial y}=x^2\cos\pi y-2ye^{-z}$ $\frac{\partial\varphi}{\partial z}=y^2e^{-z}$ 可解得

$\varphi=\frac{1}{\pi}x^2\sin\pi y-y^2e^{-z}$

以下三者等價

1. F is conservative ($F=\nabla\varphi$)
2. $\oint_c F\bullet dr=0$ $F\bullet dr=\frac{\partial\varphi}{\partial x}dx+\frac{\partial\varphi}{\partial y}dy+\frac{\partial\varphi}{\partial z}dz=d\varphi$

   $\oint_c F\bullet dr=\int_a^b\frac{d\varphi(r(t))}{dt}dt=\varphi(r(b))-\varphi(r(a))$
3. $\int F\bullet dr$ is independent of the path

A vector field F(x,y)=(M,N) is conservative

考慮其 differential form $\omega=Mdx+Ndy$ 則 $d\omega=(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y})dx\wedge dy$

Green Theorem

R 是單連通的區域 C 是其逆時針方向的封閉邊界

$$\int_C Mdx+Ndy=\iint_R(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y})dxdy$$