解 $x^3y''' + 3x^2y'' + 6xy' - 6y = x^4 \ln x$

齊次解 找 characteristic equation

Let $y = x^m$

$m(m-1)(m-2) + 3m(m-1) + 6m - 6 = 0$

$m^3 + 5m - 6 = 0$

$(m-1)(m^2 + m + 6) = 0, m = -1, \frac{-1 \pm \sqrt{23}i}{2}$

$y_h = c_1 x + x^{-\frac{1}{2}}[c_2 \cos(\frac{\sqrt{23}}{2} \ln x) + c_3 \sin(\frac{\sqrt{23}}{2} \ln x)]$

尋找特別解

猜測 $y_p = x^4(A \ln x + B)$

…

$A = \frac{1}{78}, B = -\frac{53}{6084}$

General solution $y = y_h + y_p = ...$