

解 $y'' + 2y' + y = e^{-x} \ln x$

Characteristic equation $\lambda^2 + 2\lambda + 1 = 0, \lambda = -1$

齊次解 $y_h = Ae^{-x} + Bxe^{-x}$

$$W(e^{-x}, xe^{-x}) = \begin{vmatrix} e^{-x} & xe^{-x} \\ -e^{-x} & (1-x)e^{-x} \end{vmatrix} = e^{-2x}$$

$$y_p = -y_1 \int \frac{y_2 r(x) / a_0(x)}{W(y_1, y_2)} dx + y_2 \int \frac{y_1 r(x) / a_0(x)}{W(y_1, y_2)} dx = -e^{-x} \int x \ln x dx + xe^{-x} \int \ln x dx$$
$$= \frac{1}{4} x^2 e^{-x} (2 \ln x - 1)$$

General solution $y = y_h + y_p$