§ Exact form

M dx + N d y=0 is exact $\Leftrightarrow M_y = N_x$

例 $(x^2-2y)dx+(y^2-2x)dy=0$

$\omega = Mdx + Ndy$ ，then $d\omega = (N_x - M_y)dx \wedge dy$

If $N_x - M_y = 0$ then $d\omega = 0$ $\exists\varphi$ $\omega = d\varphi$ 。The general solution I $\varphi = c$

$$\begin{cases} \dfrac{\partial\varphi}{\partial x} = x^2 - 2y \\ \dfrac{\partial\varphi}{\partial y} = y^2 - 2x \end{cases}$$

It easy to find $\varphi(x,y) = \frac{1}{3}x^3 - 2xy + \frac{1}{3}y^3$

The general solution is $\frac{1}{3}x^3 - 2xy + \frac{1}{3}y^3 = c$

Exercises

(1) $(e^x \sin y - 2y\sin x)dx + (e^x \cos y + 2\cos x)dy = 0$

$e^x \sin y + 2y\cos x = c$ or y=0

(2) $(x+\sin y)dx + (x\cos y - 2y)dy = 0$

$\frac{1}{2}x^2 + x\sin y - y^2 = c$

§ Green theorem

C is a positive oriented，piecewise smooth，simple closed curve in a plane，then

$$\int_C Mdx + Ndy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dxdy$$

例

$F(x,y) = \left(y^3, x^3 + 3xy^2\right)$

C is a circle with radius =3，F 沿 C 作功

$dW = F \bullet ds, ds = (dx, dy)$

$$W = \int_C y^3 dx + \left(x^3 + 3xy^2\right)dy$$

$$= \iint_R 3x^2 dxdy = \ldots = \frac{243\pi}{4}$$