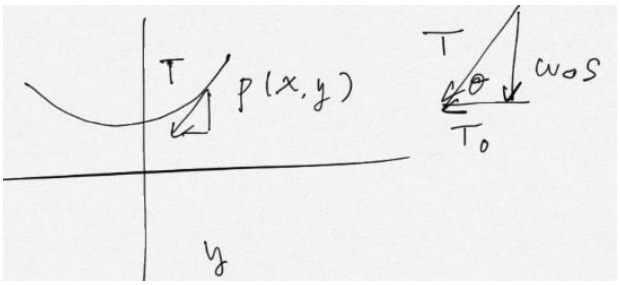
§ Catenary

https://www.pinterest.com/pin/994662267687225561/



s:$\overset{\frown}{AP}$長

ω_0 :每單位長的重量

$T_0 = T\cos\theta, \omega_0 s = T\sin\theta$

$$\frac{dy}{dx} = \tan\theta = \frac{\omega_0 s}{T_0} = as, a = \frac{\omega_0}{T_0}$$

$$ds^2 = dx^2 + dy^2$$

$$\frac{d^2y}{dx^2} = a\frac{ds}{dx} = a\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

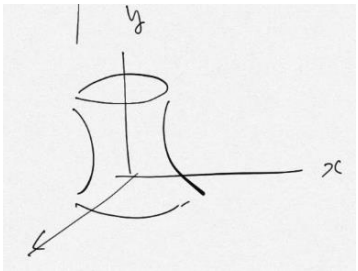
Let $p = \frac{dy}{dx}, then\ \frac{dp}{dx} = a\sqrt{1+p^2}$

$\frac{dp}{\sqrt{1+p^2}} = adx$,查表 $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + c$

$$\ln\left(\sqrt{1+p^2}\right) = ax + c, c = 0\ for\ x = 0 \Rightarrow \tan\theta = 0$$

$$e^{ax} = \sqrt{1+p^2} + p, e^{-ax} = \sqrt{1+p^2} - p, \therefore p = \frac{e^{ax} - e^{-ax}}{2}$$

$$y = \frac{e^{ax} + e^{-ax}}{2a} = \frac{1}{2}\cosh(ax)$$



catenoid

$$x = a\cosh\left(\frac{v}{a}\right)\cos u$$

$$y = a\cosh\left(\frac{v}{a}\right)\sin u$$

z=v