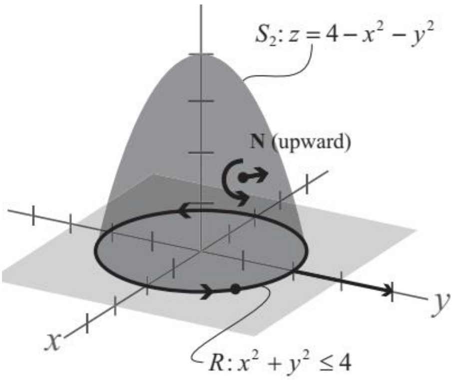
Lesson 36 Stokes Theorem and Maxwell's Equations

- Let S be an oriented surface with unit normal $\mathbf{N}$, and let C be a closed curve bounding the surface. Let $\mathbf{F}(x, y, z)$ be a vector field whose component functions have continuous first partial derivatives. Then, **Stokes's theorem** states that

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \iint_S (\text{curl } \mathbf{F}) \cdot \mathbf{N} \, dS.$$

Example



Let S be the portion of the paraboloid $z = 4 - x^2 - y^2$ above the xy-plane。Let C be its boundary，oriented counterclockwise。Verify Stokes's theorem for the function $F(x, y, z) = (2z, x, y^2)$

Exercises

3. Use Stokes's theorem to evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y, z) = xyz\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ and S is the surface $z = x^2$, $0 \leq x \leq 3$, $0 \leq y \leq 3$. Assume that $\mathbf{N}$ is the downward unit normal to the surface.

4. Assume that the motion of a liquid in a cylindrical container of radius 1 is described by the velocity field $\mathbf{F}(x, y, z) = \mathbf{i} + \mathbf{j} - 2\mathbf{k}$. Find $\iint_S (\text{curl } \mathbf{F}) \cdot \mathbf{N} \, dS$, where S is the upper surface of the cylindrical container.

5. Assume that the motion of a liquid in a cylindrical container of radius 2 is described by the velocity field $\mathbf{F}(x, y, z) = -y\sqrt{x^2 + y^2}\,\mathbf{i} + x\sqrt{x^2 + y^2}\,\mathbf{j}$. Find $\iint_S (\text{curl } \mathbf{F}) \cdot \mathbf{N} \, dS$, where S is the upper surface of the cylindrical container.