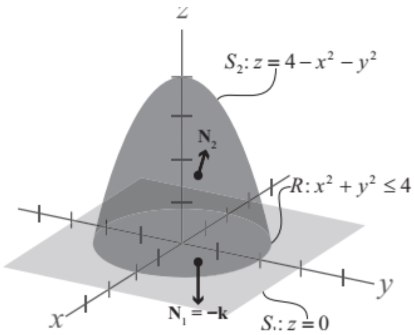
Lesson 35 Divergence Theorem

- Let Q be a solid region in space bounded by the surface S oriented outward by a unit normal $\mathbf{N}$.
  Let $\mathbf{F}(x, y, z)$ be a vector field whose component functions have continuous first partial derivatives.
  The **divergence theorem** states that

$$\iint_S \mathbf{F} \cdot \mathbf{N}\, dS = \iiint_Q \operatorname{div} \mathbf{F}\, dV.$$

Example



Let Q be the solid region between the paraboloid $z = 4 - x^2 - y^2$ and the xy-plane ◦ Verify the divergence theorem for the function $F(x, y, z) = (2z, x, y^2)$

Exercises

3. Use a graphing utility to verify the integration $\int_{-2}^{2} \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \left(4xz + 2xy + y^2\right) dx\, dy = 4\pi$, where $z = 4 - x^2 - y^2$.

4. Use the divergence theorem to evaluate the flux integral $\iint_S \mathbf{F} \cdot \mathbf{N}\, dS$, where $\mathbf{F}(x, y, z) = x^2\mathbf{i} + y^2\mathbf{j} + z^2\mathbf{k}$ and S is the cube bounded by the planes $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.

5. Use the divergence theorem to evaluate the flux integral $\iint_S \mathbf{F} \cdot \mathbf{N}\, dS$, where $\mathbf{F}(x, y, z) = x\mathbf{i} + y^2\mathbf{j} - z\mathbf{k}$ and S is the surface bounded by $x^2 + y^2 = 25$, $z = 0$, and $z = 7$.

6. For the constant vector field $\mathbf{F}(x, y, z) = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$, verify that $\iint_S \mathbf{F} \cdot \mathbf{N}\, dS = 0$, where V is the volume of the closed surface S.