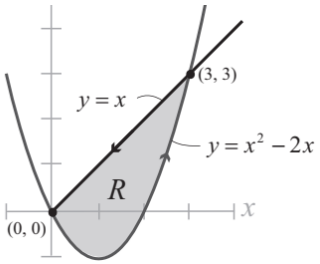
Lesson 32 Applications of Green's Theorem

- **Green's theorem**: Let C be a piecewise smooth closed curve, oriented counterclockwise. The curve is traversed once with the simply connected region R on its left. Then,

$$\int_C M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dA.$$

Example



$\frac{9}{2}$

Use Green's theorem to evaluate the integral $\int_C (y-x)\,dx + (2x-y)\,dy$, where C is the boundary of the region between the graphs of $y = x$ and $y = x^2 - 2x$.

Exercises

7. Prove that $\int_C f(x)\,dx + g(y)\,dy = 0$ if f and g are differentiable functions and C is a piecewise smooth simple closed path.