Lesson 28 Curl，Divergence，Line Integrals

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$f(x,y)=x^2y-\frac{1}{2}y^2$ ， $\nabla f=(2xy,x^2-y)$

$$curlF(x,y,z)=\nabla\times F(x,y,z)=\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}$$

$$divF(x,y,z)=\nabla\bullet F(x,y,z)=\frac{\partial P}{\partial x}+\frac{\partial Q}{\partial y}+\frac{\partial R}{\partial z}$$

Example

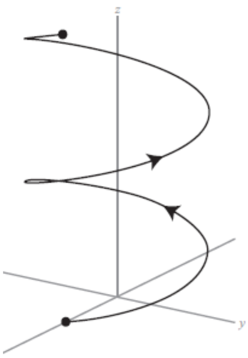
Evaluate the line integral $\int_C (x^2-y+3z)\,ds$, where C is the line segment given by $\mathbf{r}(t)=t\mathbf{i}+2t\mathbf{j}+t\mathbf{k}$, $0\le t\le 1$.

$ds^2=(dx)^2+(dy)^2+(dz)^2$ ，其中 x(t)=t，y(t)=2t，z(t)=t

$$ds=\sqrt{(\frac{dx}{dt})^2+(\frac{dy}{dt})^2+(\frac{dz}{dt})^2}\,dt=\sqrt{6}dt$$

$$\int_C (x^2-y+3z)ds=\int_0^1 (t^2-2t+3t)\sqrt{6}dt=\frac{5\sqrt{6}}{6}$$

Example



circular helix $r(t)=(\cos t,\sin t,t), 0\le t\le 6\pi$

the density of the spring is $\rho(x,y,z)=1+z$

Calculus the mass of the spring

$r'(t)=(-\sin t,\cos t,1)$

$$ds=|r'(t)|dt=\sqrt{2}dt$$

$$\int_C (1+z)ds=\int_0^{6\pi}(1+t)\sqrt{2}dt=6\sqrt{2}\pi(3\pi+1)$$

Exercises

7. Find the divergence of the vector field $\mathbf{F}(x,y,z)=\sin x\mathbf{i}+\cos y\mathbf{j}+z^2\mathbf{k}$.

8. Evaluate the line integral $\int_C xy\,ds$, where C is the path $\mathbf{r}(t)=4t\mathbf{i}+3t\mathbf{j}$, $0\le t\le 1$.

9. Evaluate the line integral $\int_C (x^2+y^2+z^2)\,ds$, where C is the path $\mathbf{r}(t)=\sin t\mathbf{i}+\cos t\mathbf{j}+2\mathbf{k}$, $0\le t\le \frac{\pi}{2}$.

10. Verify that $\int_0^{6\pi}(1+t)\sqrt{2}\,dt=6\pi\sqrt{2}(3\pi+1)$.