

Lesson 27 Vector Fields---Velocity 、 Gravity 、 Electricity

- A vector field \mathbf{F} is **conservative** if there exists a differentiable function f (the **potential**) such that $\mathbf{F} = \nabla f$.
- Theorem: Let M and N have continuous first partial derivatives on an open disk R . The vector field given by $\mathbf{F}(x, y) = M\mathbf{i} + N\mathbf{j}$ is conservative if and only if $\frac{\partial N}{\partial x} = \frac{\partial M}{\partial y}$.

Example

The vector field $\mathbf{F}(x, y) = (10x + 3y)\mathbf{i} + (3x + 2y)\mathbf{j}$ is conservative with potential $f(x, y) = 5x^2 + 3xy + y^2$ because $\mathbf{F} = \nabla f$.

Exercises

7. Find the conservative vector field for the potential function $f(x, y, z) = z + ye^{x^2}$.
8. Find a potential function for the conservative vector field $\mathbf{F}(x, y) = y\mathbf{i} + x\mathbf{j}$.
9. Find a potential function for the conservative vector field $\mathbf{F}(x, y) = \frac{x}{x^2 + y^2}\mathbf{i} + \frac{y}{x^2 + y^2}\mathbf{j}$.
10. Find a potential function for the conservative vector field $\mathbf{F}(x, y) = 3x^2y^2\mathbf{i} + 2x^3y\mathbf{j}$.