Lesson 25

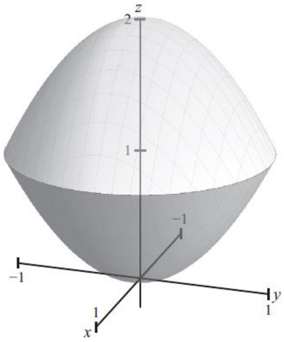
§ 圓柱座標系

Let $P = (x, y, z)$ be a point in space. Its **cylindrical coordinates** are (r, θ, z), where (r, θ) are the polar coordinates of the projection of the point onto the xy-plane. The z coordinate is the same.

$$x = r\cos\theta, y = r\sin\theta, z = z$$

Example

Set up the triple integral for the volume of the solid region bounded below by the surface $z = x^2 + y^2$ and above by $z = 2 - x^2 - y^2$.



$$V = \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{x^2+y^2}^{2-x^2-y^2} dzdydx =$$

用圓柱座標(r, θ, z)

$$x = r\cos\theta, y = r\sin\theta, z = z$$

$$dV = dxdydz = rdzdrd\theta$$

$$V = \iiint_Q dV = \int_0^{2\pi}\int_0^1\int_{r^2}^{2-r^2} rdzdrd\theta = \ldots = \pi$$

Exercise

6. Find an equation in rectangular coordinates for the cylindrical equation $r = 2\sin\theta$.

7. Verify that $V = 2\int_0^{2\pi}\int_{R_2}^{R_1}\int_0^{\sqrt{R_1^2 - r^2}} r\,dz\,dr\,d\theta = \frac{4\pi}{3}\left(R_1^2 - R_2^2\right)^{3/2}$.

8. Convert the integral $\int_{-2}^{2}\int_{\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{x^2+y^2}^{4} x\,dz\,dy\,dx$ to cylindrical coordinates.

9. Set up the triple integral in cylindrical coordinates for the volume of the solid bounded above by $z = 2x$ and below by $z = 2x^2 + 2y^2$.

10. Set up the triple integral in cylindrical coordinates for the volume of the solid inside the sphere $x^2 + y^2 + z^2 = 4$ and above the upper nappe of the cone $z^2 = x^2 + y^2$.