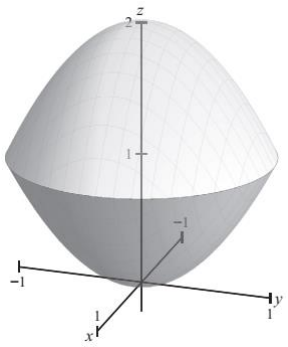
Lesson 24 Triple Integrals and Applications

Example



Set up the triple integral for the volume of the solid region bounded below by the surface $z = x^2 + y^2$ and above by $z = 2 - x^2 - y^2$

$$\begin{cases} z = x^2 + y^2 \\ z = 2 - x^2 - y^2 \end{cases} \Rightarrow x^2 + y^2 = 1$$

$$\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{x^2+y^2}^{2-x^2-y^2} dz\,dy\,dx = \qquad \pi$$

Exercises

8. Set up the integral for the mass of the solid bounded by $3x + 3y + 5z = 15$, $x = 0$, $y = 0$, and $z = 0$ if the density is $\rho(x, y, z) = 3y$.

9. Rewrite the iterated integral $\int_0^1 \int_{-1}^{0} \int_0^{y^2} dz\,dy\,dx$ using the order $dy\,dz\,dx$.

10. Rewrite the iterated integral $\int_0^4 \int_0^{(4-x)/2} \int_0^{(12-3x-6y)/4} dz\,dy\,dx$ using the order $dy\,dx\,dz$.