Lesson 22 Centers of Mass for variable Density

- If the planar lamina given by the region R has variable density $\rho(x, y)$, then the **mass** is

$$m = \iint_R \rho(x,y)\,dA .$$

- The **moments of mass** with respect to the x- and y-axes are

$$M_x = \iint_R y\rho(x,y)\,dA,\ M_y = \iint_R x\rho(x,y)\,dA .$$

- If m is the mass of the lamina, the **center of mass** is $(\bar{x}, \bar{y}) = \left(\frac{M_y}{m}, \frac{M_x}{m}\right)$.

Lamina 葉片

Example

Find the center of mass of the lamina corresponding to the parabolic region $0 \le y \le 4 - x^2$ if the density at the point (x, y) is constant, $\rho(x, y) = 1$.

$$m = \int_{-2}^{2}\int_{0}^{4-x^2} 1dydx = \frac{32}{3}$$

$$M_x = \int_{-2}^{2}\int_{0}^{4-x^2} ydydx = \frac{256}{15} \text{，} M_y = \int_{-2}^{2}\int_{0}^{4-x^2} xdydx = 0$$

The center of mass is $(\bar{x}, \bar{y}) = (\frac{M_y}{m}, \frac{M_x}{m}) = (0, \frac{8}{5})$

Exercises

6. Find the mass and center of mass of the lamina bounded by $y = x^2$, $y = 0$, and $x = 2$ if the density is $\rho(x, y) = 3xy$.

7. Find the mass and center of mass of the lamina bounded by $x^2 + y^2 = 16$, $0 \le x$, and $0 \le y$ if the density is $\rho(x, y) = 3(x^2 + y^2)$.