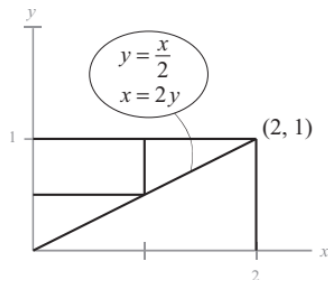


Lesson 20 Double Integrals and Volume

Example



Find the volume under the surface $z = \sin^2 y$ and above the region bounded by $y = \frac{x}{2}$, $x=0$, and $y=1$

$$V = \int_0^2 \int_{\frac{x}{2}}^1 \sin^2 y \, dy \, dx = \int_0^1 \int_0^{2y} \sin^2 y \, dx \, dy = -\cos 1 + 1$$

Exercises

- Evaluate the iterated integral $\int_0^1 \int_{\frac{y}{2}}^{\frac{y}{2}} e^{-x^2} \, dx \, dy$ by switching the order of integration.
- Evaluate the iterated integral $\int_0^2 \int_{\frac{x^2}{2}}^2 \sqrt{y} \cos y \, dy \, dx$ by switching the order of integration.
- Find the average value of the function $f(x, y) = x$ over the rectangle with vertices $(0, 0)$, $(4, 0)$, $(4, 2)$, and $(0, 2)$.
- Find the average value of the function $f(x, y) = \sin(x + y)$ over the rectangle with vertices $(0, 0)$, $(\pi, 0)$, (π, π) , and $(0, \pi)$.
- Why is the expression $\int_0^2 \int_0^y (x + y) \, dy \, dx$ invalid?