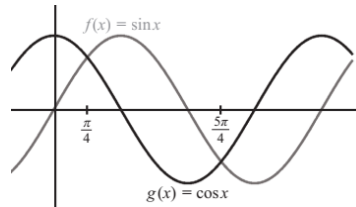


### § Iterated Integral(逐次積分)

Iterated integrals are of the form  $\int_c^d \left[ \int_{h_1(y)}^{h_2(y)} f(x, y) dx \right] dy$  and  $\int_a^b \left[ \int_{g_1(x)}^{g_2(x)} f(x, y) dy \right] dx$ .

Example 1 Calculus the iterated integral  $\int_2^4 \left( \int_1^x 2xy dy \right) dx$  54

Example 2



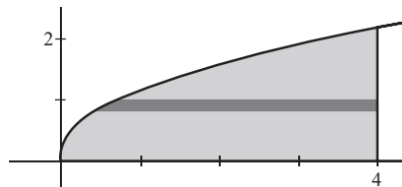
Find the area of the region bounded by the graph of

$f(x) = \sin x$ ,  $g(x) = \cos x$  between  $x = \frac{\pi}{4}$  and  $x = \frac{5\pi}{4}$

$$\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \int_{\cos x}^{\sin x} dy dx = 2\sqrt{2}$$

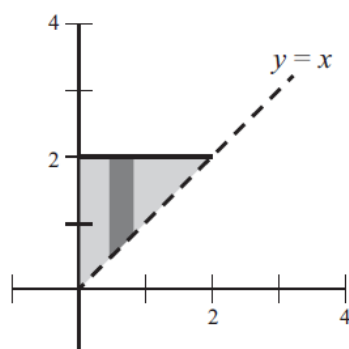
Example 3

Sketch the region whose area is represented by the iterated integral  $\int_0^2 \int_{y^2}^4 dx dy$ .



Example 4

Sketch the region of integration represented by the iterated integral  $\int_0^2 \int_x^2 e^{-y^2} dy dx$



$$\int_0^2 \int_x^2 e^{-y^2} dy dx = \int_0^2 \int_0^y e^{-y^2} dx dy = \int_0^2 ye^{-y^2} dy$$

$$\text{Let } u = e^{-y^2}, du = -2ye^{-y^2} dy$$

$$\int_0^2 ye^{-y^2} dy = \left( -\frac{1}{2} \right) e^{-y^2} \Big|_0^2 = \frac{1}{2} \left( 1 - \frac{1}{e^4} \right)$$

Exercise 10 Evaluate the iterated integral  $\int_0^2 \int_{y^2}^4 \sqrt{x} \sin x dx dy$

$$\int_0^2 \int_{y^2}^4 \sqrt{x} \sin x dx dy = \int_0^4 \int_0^{\sqrt{x}} \sqrt{x} \sin x dy dx = \int_0^4 x \sin x dx$$

Let  $u=x$ ,  $v=-\cos x$ ,

$$\int x \sin x dx = -x \cos x - \int (-\cos x) dx = -x \cos x + \sin x$$

所以原式= $-4\cos 4 + \sin 4$