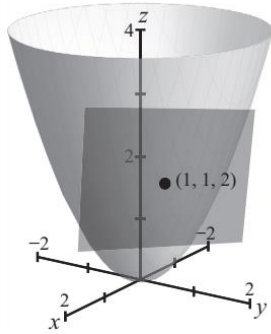Lesson 16 tangent plane and level surface

Example

Find the equation of the tangent plane to the surface $z = x^2 + y^2$ at

the point $(1, 1, 2)$.

解 1

Define $F(x, y, z) = f(x, y) - z = x^2 + y^2 - z$，and

consider the level surface F(x,y,z)=0

The gradient is $\nabla F = (2x, 2y, -1)$

$\nabla F(1,1,2) = (2, 2, -1)$ is the normal vector at (1,1,2)

Then the tangent plane is 2x+2y-z=2

解 2

$X = (x, y, x^2 + y^2), X_x = (1, 0, 2x), X_y = (0, 1, 2y)$

$X_x \times X_y = (-2x, -2y, 1)$ is the normal vector

Exercises

8. Find an equation of the tangent plane to the surface $x^2 + 4y^2 + z^2 = 36$ at the point $(2, -2, 4)$.

9. Find the point(s) on the surface $z = 3 - x^2 - y^2 + 6y$ at which the tangent plane is horizontal.

10. Find the point(s) on the surface $z = xy + \dfrac{1}{x} + \dfrac{1}{y}$ at which the tangent plane is horizontal.