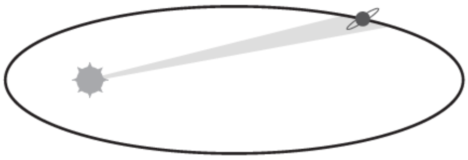
Lesson 14 Kepler's laws---The Calculus of Orbits

§ Kepler's laws



(1) 行星的軌跡是一橢圓，太陽在焦點。

(2) 連接太陽與行星的線在相同時間掃出相等面積

(3) $\frac{a^3}{T^2} = \text{constant}$

Kepler's famous laws of planetary motion were announced by the German astronomer and mathematician Johannes Kepler (1571–1630). His three laws were based on a 20-year study of astronomical data compiled by the Danish astronomer Tycho Brahe. Isaac Newton later used calculus to derive these laws from basic laws of physics. In this lesson, we will study these three laws and use our calculus skills to prove the second law.

太陽在原點，行星位置$P(r,\theta)$

$\omega = \frac{d\theta}{dt}$稱為角速度，$m \cdot r^2\omega$稱為角動量，在守恆系力距恆為零$\Leftrightarrow$角動量為常數(力距=角動量對時間的變化率)

$$\frac{dA}{dt} = \frac{\frac{1}{2}r^2 d\theta}{dt} = \frac{1}{2}r^2\frac{d\theta}{dt} = \frac{1}{2}r^2\omega = \text{constant}$$

表示$\overline{OP} \times F \equiv 0$，作用力的方向在$\overline{OP}$直線上 即行星恆受一正對太陽的力所作用。

$\frac{(x-c)^2}{a^2} + \frac{y^2}{b^2} = 1$，let $x = r\cos\theta, y = r\sin\theta$ 可推出$r = \frac{b^2}{a - c\cos\theta} = \frac{b^2}{a(1 - e\cos\theta)}$

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Example

1. Show that a given planet moves in a plane。
2. Prove Kepler's second law

Exercises

1. Halley's Comet has an elliptical orbit with the Sun at one focus and has an eccentricity of $e \approx 0.967$. The length $(2a)$ of the major axis of the orbit is approximately 35.88 astronomical units (AU). (An astronomical unit is defined as the mean distance between Earth and the Sun, 93 million miles.) A polar equation for the orbit is

$$r = \frac{ed}{1 + e\sin\theta}.$$

Find the value of d. Then, use the fact that $c = ea$ is the distance between the focus and the center to determine how close the comet comes to the Sun.

2. The asteroid Apollo has a period of 661 Earth days, and its orbit is approximated by the ellipse

$$r = \frac{1}{1 + \left(5/9\right)\cos\theta} = \frac{9}{9 + 5\cos\theta},$$

where r is measured in astronomical units. The area of this ellipse is approximately 5.46507. Use a graphing utility to approximate the time it takes Apollo to move from the position given by $\theta = -\pi/2$ to $\theta = \pi/2$.