APCalculus2016

Question 1

t (hours)	0	1	3	6	8
$R(t)$ (liters / hour)	1340	1190	950	740	700

Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \le t \le 8$, where t is measured in hours. Water is removed from the tank at a rate modeled by $R(t)$ liters per hour, where R is differentiable and decreasing on $0 \le t \le 8$. Selected values of $R(t)$ are shown in the table above. At time $t = 0$, there are 50,000 liters of water in the tank.

(a) Estimate $R'(2)$. Show the work that leads to your answer. Indicate units of measure.

(b) Use a left Riemann sum with the four subintervals indicated by the table to estimate the total amount of water removed from the tank during the 8 hours. Is this an overestimate or an underestimate of the total amount of water removed? Give a reason for your answer.

(c) Use your answer from part (b) to find an estimate of the total amount of water in the tank, to the nearest liter, at the end of 8 hours.

(d) For $0 \le t \le 8$, is there a time t when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain why or why not.

ANS

(a) mean value theorem $R'(2) \approx \dfrac{R(3)-R(1)}{3-1} = -120$ (liters/hr^2)

(b) Left Riemann sum $\int_0^8 R(t)dt \approx 8050$ liters

(c) Total $\approx 5000 + \int_0^8 W(t)dt - 8050 \approx 49786$ liters

$W(0) - R(0) > 0$, $W(8) - R(8) < 0$, and $W(t) - R(t)$ is continuous.

Therefore, the Intermediate Value Theorem guarantees at least one

(d) time t, $0 < t < 8$, for which $W(t) - R(t) = 0$, or $W(t) = R(t)$.

Question 2

For $t \geq 0$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = 1 + 2\sin\left(\frac{t^2}{2}\right)$. The particle is at position $x = 2$ at time $t = 4$.

(a) At time $t = 4$, is the particle speeding up or slowing down?

(b) Find all times t in the interval $0 < t < 3$ when the particle changes direction. Justify your answer.

(c) Find the position of the particle at time $t = 0$.

(d) Find the total distance the particle travels from time $t = 0$ to time $t = 3$.

ANS

$$v(4) = 2.978716 > 0$$
$$v'(4) = -1.164000 < 0$$

(a) The particle is slowing down since the velocity and acceleration have different signs.

(b) v(t)=0 t=2.707

(c) $x(0) = x(4) + \int_4^0 v(t)dt = -3.815$

(d) Distance= $\int_0^3 |v(t)|\, dt = 5.301$

Question 3

The figure above shows the graph of the piecewise-linear function f. For $-4 \le x \le 12$, the function g is defined by

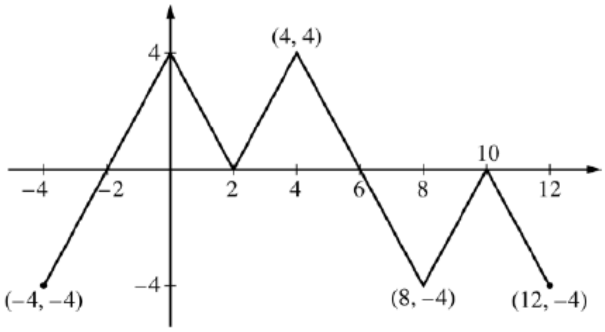
$g(x) = \int_2^x f(t)\,dt.$

(a) Does g have a relative minimum, a relative maximum, or neither at $x = 10$? Justify your answer.

(b) Does the graph of g have a point of inflection at $x = 4$? Justify your answer.

(c) Find the absolute minimum value and the absolute maximum value of g on the interval $-4 \le x \le 12$. Justify your answers.

(d) For $-4 \le x \le 12$, find all intervals for which $g(x) \le 0$.



Graph of f

ANS

(a) 微積分基本定理 $g'(x) = f(x)$

g has neither a relative minimum nor a relative maximum

(b) The graph of g has a point of inflection at $x = 4$ since $g'(x) = f(x)$ is increasing for $2 \le x \le 4$ and decreasing for $4 \le x \le 8$.

(c) the absolute minimum is g(-2)=-8; and the absolute maximum is g(6)=8

(d) $-4 \le x \le 2$ or $10 \le x \le 12$

Question 4

Consider the differential equation $\frac{dy}{dx} = \frac{y^2}{x-1}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.

(b) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(2) = 3$. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 2$. Use your equation to approximate $f(2.1)$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(2) = 3$.

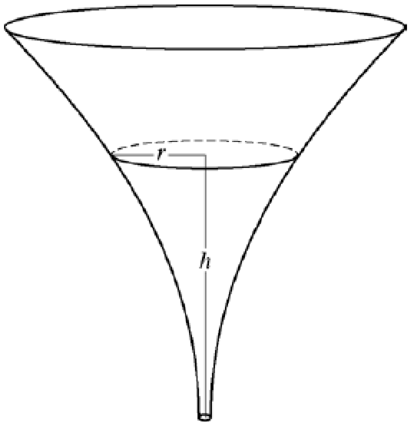
ANS

(a)

(b) $\frac{dy}{dx}\Big|_{(2,3)} = 9$ ，y=9x-15;f(2,1)$\approx$3.9

(c)微分方程 分離變數法 $y = \frac{3}{1-3\ln(x-1)}$ for $1 < x < 1 + e^{\frac{1}{3}}$

Question 5



The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height h, the radius of the funnel is given by $r = \frac{1}{20}\left(3 + h^2\right)$, where $0 \le h \le 10$. The units of r and h are inches.

(a) Find the average value of the radius of the funnel.

(b) Find the volume of the funnel.

(c) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is $h = 3$ inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{5}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?

ANS

(a) $\frac{1}{10}\int_0^{10}\frac{1}{20}(3+h^2)dh = \frac{109}{60}$ in

(b)旋轉體體積 $V = \pi\int_0^{10}(\frac{1}{20}(3+h^2))^2 dh = \frac{2209\pi}{40} in^3$

(c)chain rule $\frac{dh}{dt} = -\frac{2}{3} in/\sec$

Question 6

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	-6	3	2	8
2	2	-2	-3	0
3	8	7	6	2
6	4	5	3	-1

The functions f and g have continuous second derivatives. The table above gives values of the functions and their derivatives at selected values of x.

(a) Let $k(x) = f(g(x))$. Write an equation for the line tangent to the graph of k at $x = 3$.

(b) Let $h(x) = \frac{g(x)}{f(x)}$. Find $h'(1)$.

(c) Evaluate $\int_1^3 f''(2x)\, dx$.

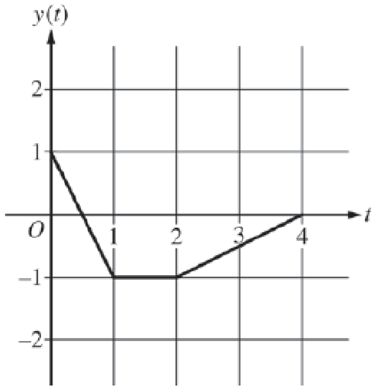
ANS

(a) y=10x-26

(b) $h'(1) = -\frac{3}{2}$

(c) $\frac{7}{2}$

BC



2. At time t, the position of a particle moving in the xy-plane is given by the parametric functions $(x(t), y(t))$, where $\frac{dx}{dt} = t^2 + \sin(3t^2)$. The graph of y, consisting of three line segments, is shown in the figure above. At $t = 0$, the particle is at position $(5, 1)$.

   (a) Find the position of the particle at $t = 3$.

   (b) Find the slope of the line tangent to the path of the particle at $t = 3$.

   (c) Find the speed of the particle at $t = 3$.

   (d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

ANS

(a)(x(3),y(3))=(14.377,-0.5)

(b)slope=$\frac{y'(3)}{x'(3)}$=0.05

(c)9.969

(d)distance=$\int_0^2 \sqrt{(x'(t))^2 + (y'(t))^2}\,dt$ =4.350

4. Consider the differential equation $\frac{dy}{dx} = x^2 - \frac{1}{2}y$.

(a) Find $\frac{d^2y}{dx^2}$ in terms of x and y.

(b) Let $y = f(x)$ be the particular solution to the given differential equation whose graph passes through the point $(-2, 8)$. Does the graph of f have a relative minimum, a relative maximum, or neither at the point $(-2, 8)$? Justify your answer.

(c) Let $y = g(x)$ be the particular solution to the given differential equation with $g(-1) = 2$. Find $\lim_{x \to -1}\left(\frac{g(x) - 2}{3(x + 1)^2}\right)$. Show the work that leads to your answer.

(d) Let $y = h(x)$ be the particular solution to the given differential equation with $h(0) = 2$. Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $h(1)$.

ANS

(a) $\frac{d^2y}{dx^2} = 2x - \frac{1}{2}x^2 + \frac{1}{4}y$

(b)f has a relative maximum at (-2,8)

(c) $L'Hospital$ rule $-\frac{1}{3}$

(d)Euler method $h(1) \approx \frac{5}{4}$

6. The function f has a Taylor series about $x = 1$ that converges to $f(x)$ for all x in the interval of convergence. It is known that $f(1) = 1$, $f'(1) = -\frac{1}{2}$, and the nth derivative of f at $x = 1$ is given by $f^{(n)}(1) = (-1)^n \frac{(n-1)!}{2^n}$ for $n \geq 2$.

(a) Write the first four nonzero terms and the general term of the Taylor series for f about $x = 1$.

(b) The Taylor series for f about $x = 1$ has a radius of convergence of 2. Find the interval of convergence. Show the work that leads to your answer.

(c) The Taylor series for f about $x = 1$ can be used to represent $f(1.2)$ as an alternating series. Use the first three nonzero terms of the alternating series to approximate $f(1.2)$.

(d) Show that the approximation found in part (c) is within 0.001 of the exact value of $f(1.2)$.

ANS

(a)
$$f(x) = 1 - \frac{1}{2}(x-1) + \frac{1}{2^2 \cdot 2}(x-1)^2 - \frac{1}{2^3 \cdot 3}(x-1)^3 + \cdots + \frac{(-1)^n}{2^n \cdot n}(x-1)^n + \cdots$$

(b) $-1 < x \leq 3$

(c) $f(1.2) \approx 1 - \frac{1}{2} \times 0.2 + \frac{1}{8} \times 0.2^2 = 0.905$

(d)
$$|f(1.2) - T_2(1.2)| \leq \left| \frac{-1}{2^3 \cdot 3}(0.2)^3 \right| = \frac{1}{3000} \leq 0.001$$