APCalculus2012AB and BC

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

1. The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is 55°F. The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

(a) Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

(b) Use the data in the table to evaluate $\int_0^{20} W'(t)\,dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t)\,dt$ in the context of this problem.

(c) For $0 \le t \le 20$, the average temperature of the water in the tub is $\frac{1}{20}\int_0^{20} W(t)\,dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W(t)\,dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

(d) For $20 \le t \le 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t}\cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?

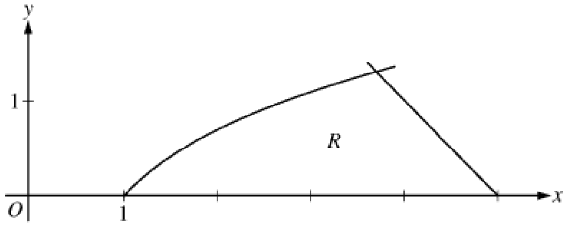
(a) The water temperature is increasing at a rate of approximately 1017^{0}Fper minute at time t=12 minutes

(b) The water has warmed by 16^{0}F over the interval from t-0 to t=20 minutes

(c) This approximation is an underestimate，because a left Riemann sum is used and the function W is strictly increasing。

(d) 73.043

2\. Let R be the region in the first quadrant bounded by the x-axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure above.



(a) Find the area of R.

(b) Region R is the base of a solid. For the solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

(c) The horizontal line $y = k$ divides R into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k.

ANS

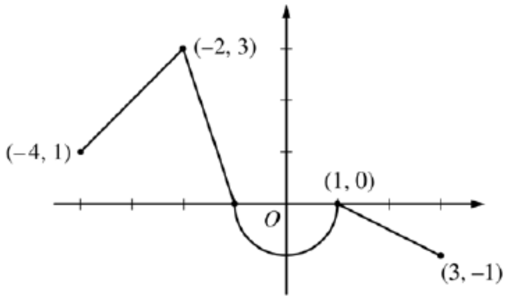
(a)2.986

(a) 體積 $dV = y^2 dx$ ，則 $V = \int y^2 dx$

(c) $\int_0^k (5 - y - e^y) dy = \frac{1}{2} \times 2.986$

3.

Let f be the continuous function defined on $[-4, 3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let g be the function given by $g(x) = \int_1^x f(t)\, dt$.



Graph of f

(a) Find the values of $g(2)$ and $g(-2)$.

(b) For each of $g'(-3)$ and $g''(-3)$, find the value or state that it does not exist.

(c) Find the x-coordinate of each point at which the graph of g has a horizontal tangent line. For each of these points, determine whether g has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.

(d) For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

ANS

(a) $g(2) = -\frac{1}{4}$ ， $g(-2) = \frac{\pi}{2} - \frac{3}{2}$

(b) $g'(-3) = 2$ ， $g''(-3) = 1$

(c)g 在-1 有相對極大，在 1 沒有極值

(d)g 在 x=-2,0,1 有反曲點

4. The function f is defined by $f(x) = \sqrt{25 - x^2}$ for $-5 \le x \le 5$.

(a) Find $f'(x)$.

(b) Write an equation for the line tangent to the graph of f at $x = -3$.

(c) Let g be the function defined by $g(x) = \begin{cases} f(x) & \text{for } -5 \le x \le -3 \\ x + 7 & \text{for } -3 < x \le 5. \end{cases}$

Is g continuous at $x = -3$? Use the definition of continuity to explain your answer.

(d) Find the value of $\int_0^5 x\sqrt{25 - x^2}\, dx$.

ANS

(a)

(b) $y = 4 + \frac{3}{4}(x + 3)$

(c)g 在 x=-3 連續

(d) $\frac{125}{3}$

The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

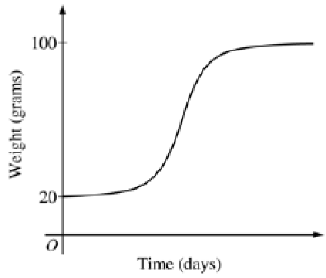
$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

5. Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

(a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.

(b) Find $\frac{d^2B}{dt^2}$ in terms of B. Use $\frac{d^2B}{dt^2}$ to explain why the graph of B cannot resemble the following graph.

(c) Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.



ANS

(a) 在 40 克時較快

(b)

(c) $B(t) = 100 - 80e^{-\frac{t}{5}}$，$t \geq 0$

6. For $0 \le t \le 12$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = \cos\left(\frac{\pi}{6}t\right)$. The particle is at position $x = -2$ at time $t = 0$.

(a) For $0 \le t \le 12$, when is the particle moving to the left?

(b) Write, but do not evaluate, an integral expression that gives the total distance traveled by the particle from time $t = 0$ to time $t = 6$.

(c) Find the acceleration of the particle at time t. Is the speed of the particle increasing, decreasing, or neither at time $t = 4$? Explain your reasoning.

(d) Find the position of the particle at time $t = 4$.

質點的運動方程式

(a) v(t)>0 表示質點向右，v(t)<0 表示質點向左運動

(b) 質點運動的總距離= $\int_0^6 |v(t)| dt$

(c) 加速度 $a(t) = v'(t)$ ，要注意方向性

(d)積分 假設位置函數為 x(t)，則 $v(t) = x'(t)$ ，所以 $x(t) = \int v(t) dt$

ANS

(a)3<t<9

(b) $\int_0^6 |v(t)| dt$

(c)速率漸增(a(4),v(4)同號)

(d) $-2 + \frac{3\sqrt{3}}{\pi}$

7. For $t \geq 0$, a particle is moving along a curve so that its position at time t is $(x(t), y(t))$. At time $t = 2$, the particle is at position $(1, 5)$. It is known that $\frac{dx}{dt} = \frac{\sqrt{t+2}}{e^t}$ and $\frac{dy}{dt} = \sin^2 t$.

(a) Is the horizontal movement of the particle to the left or to the right at time $t = 2$? Explain your answer. Find the slope of the path of the particle at time $t = 2$.

(b) Find the x-coordinate of the particle's position at time $t = 4$.

(c) Find the speed of the particle at time $t = 4$. Find the acceleration vector of the particle at time $t = 4$.

(d) Find the distance traveled by the particle from time $t = 2$ to $t = 4$.

參數方程式 質點的運動軌跡

(a)

(b)

(c)速度 velocity 是向量 $v(t) = (x'(t), y'(t))$ 速率 speed= $|v(t)| = \sqrt{(x'(t))^2 + (y'(t))^2}$

(d)質點移動的距離= $\int |v(t)|\, dt$

ANS

(a)

(b)1.253

(c)(1)0.575 (2)<-0.041,0.989>

(d)0.651

x	1	1.1	1.2	1.3	1.4
$f'(x)$	8	10	12	13	14.5

8. The function f is twice differentiable for $x > 0$ with $f(1) = 15$ and $f''(1) = 20$. Values of f', the derivative of f, are given for selected values of x in the table above.

(a) Write an equation for the line tangent to the graph of f at $x = 1$. Use this line to approximate $f(1.4)$.

(b) Use a midpoint Riemann sum with two subintervals of equal length and values from the table to approximate $\int_1^{1.4} f'(x)\,dx$. Use the approximation for $\int_1^{1.4} f'(x)\,dx$ to estimate the value of $f(1.4)$. Show the computations that lead to your answer.

(c) Use Euler's method, starting at $x = 1$ with two steps of equal size, to approximate $f(1.4)$. Show the computations that lead to your answer.

(d) Write the second-degree Taylor polynomial for f about $x = 1$. Use the Taylor polynomial to approximate $f(1.4)$.

(a) 切線方程式

(b) 黎曼和

(c) Euler 法求近似值

(d) 泰勒展開式

ANS

(a)18.2

(b)19.6

(c)(1)f(1.2)=16.6 (2)f(1.4)=19.0

(d)19.8

9. The function g has derivatives of all orders, and the Maclaurin series for g is
$$\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+3}=\frac{x}{3}-\frac{x^3}{5}+\frac{x^5}{7}-\cdots.$$

(a) Using the ratio test, determine the interval of convergence of the Maclaurin series for g.

(b) The Maclaurin series for g evaluated at $x=\frac{1}{2}$ is an alternating series whose terms decrease in absolute value to 0. The approximation for $g\left(\frac{1}{2}\right)$ using the first two nonzero terms of this series is $\frac{17}{120}$. Show that this approximation differs from $g\left(\frac{1}{2}\right)$ by less than $\frac{1}{200}$.

(c) Write the first three nonzero terms and the general term of the Maclaurin series for $g'(x)$.

(a) 比較審斂法(ratio test)求 Maclaurin 級數的收斂區間

$\left|\frac{a_{n+1}}{a_n}\right|<1$，在收斂範圍兩端要另行討論

(b)

(c)

ANS

(a) $-1\le x\le 1$

(b) $$\left|g\left(\frac{1}{2}\right)-\frac{17}{120}\right|<\frac{\left(\frac{1}{2}\right)^5}{7}=\frac{1}{224}<\frac{1}{200}$$

(c) $$g'(x)=\frac{1}{3}-\frac{3}{5}x^2+\frac{5}{7}x^4+\cdots+(-1)^n\left(\frac{2n+1}{2n+3}\right)x^{2n}+\cdots$$