2008AP Calculus BC

1. At time $t \geq 0$, a particle moving in the xy-plane has velocity vector given by $v(t) = \langle t^2, 5t \rangle$. What is the acceleration vector of the particle at time $t = 3$?

(A) $\left\langle 9, \frac{45}{2} \right\rangle$ (B) $\langle 6,5 \rangle$ (C) $\langle 2,0 \rangle$ (D) $\sqrt{306}$ (E) $\sqrt{61}$

2. $\int x e^{x^2} dx =$

(A) $\frac{1}{2} e^{x^2} + C$ (B) $e^{x^2} + C$ (C) $x e^{x^2} + C$ (D) $\frac{1}{2} e^{2x} + C$ (E) $e^{2x} + C$

3. $\lim_{x \to 0} \frac{\sin x \cos x}{x}$ is

(A) -1 (B) 0 (C) 1 (D) $\frac{\pi}{4}$ (E) nonexistent

4. Consider the series $\sum_{n=1}^{\infty} \frac{e^n}{n!}$. If the ratio test is applied to the series, which of the following inequalities results, implying that the series converges?

(A) $\lim_{n \to \infty} \frac{e}{n!} < 1$

(B) $\lim_{n \to \infty} \frac{n!}{e} < 1$

(C) $\lim_{n \to \infty} \frac{n+1}{e} < 1$

(D) $\lim_{n \to \infty} \frac{e}{n+1} < 1$

(E) $\lim_{n \to \infty} \frac{e}{(n+1)!} < 1$

5. Which of the following gives the length of the path described by the parametric equations $x=\sin\left(t^3\right)$ and $y=e^{5t}$ from $t=0$ to $t=\pi$?

(A) $\int_0^{\pi}\sqrt{\sin^2\left(t^3\right)+e^{10t}}\,dt$

(B) $\int_0^{\pi}\sqrt{\cos^2\left(t^3\right)+e^{10t}}\,dt$

(C) $\int_0^{\pi}\sqrt{9t^4\cos^2\left(t^3\right)+25e^{10t}}\,dt$

(D) $\int_0^{\pi}\sqrt{3t^2\cos\left(t^3\right)+5e^{5t}}\,dt$

(E) $\int_0^{\pi}\sqrt{\cos^2\left(3t^2\right)+e^{10t}}\,dt$

$$f(x)=\begin{cases}\dfrac{x^2-4}{x-2} & \text{if } x\neq 2\\ 1 & \text{if } x=2\end{cases}$$

6. Let f be the function defined above. Which of the following statements about f are true?

I. f has a limit at $x=2$.

II. f is continuous at $x=2$.

III. f is differentiable at $x=2$.

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III

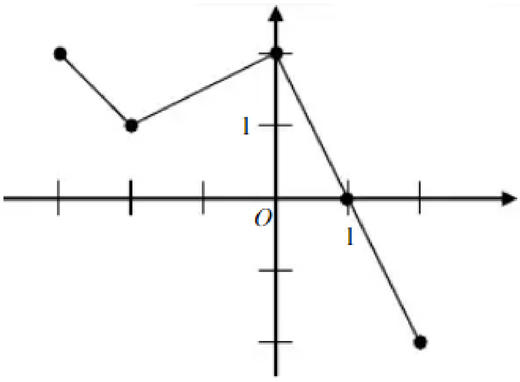
7. Given that $y(1)=-3$ and $\dfrac{dy}{dx}=2x+y$, what is the approximation for $y(2)$ if Euler's method is used with a step size of 0.5, starting at $x=1$?

(A) -5 (B) -4.25 (C) -4 (D) -3.75 (E) -3.5

x	2	3	5	8	13
$f(x)$	6	-2	-1	3	9

8. The function f is continuous on the closed interval $[2, 13]$ and has values as shown in the table above. Using the intervals $[2, 3]$, $[3, 5]$, $[5, 8]$, and $[8, 13]$, what is the approximation of $\int_2^{13} f(x)\,dx$ obtained from a left Riemann sum?

(A) 6 (B) 14 (C) 28 (D) 32 (E) 50



Graph of f

9. The graph of the piecewise linear function f is shown in the figure above. If $g(x) = \int_{-2}^{x} f(t)\,dt$, which of the following values is greatest?

(A) $g(-3)$ (B) $g(-2)$ (C) $g(0)$ (D) $g(1)$ (E) $g(2)$

10. In the xy-plane, what is the slope of the line tangent to the graph of $x^2 + xy + y^2 = 7$ at the point $(2, 1)$?

(A) $-\frac{4}{3}$ (B) $-\frac{5}{4}$ (C) -1 (D) $-\frac{4}{5}$ (E) $-\frac{3}{4}$

11. Let R be the region between the graph of $y = e^{-2x}$ and the x-axis for $x \geq 3$. The area of R is

(A) $\frac{1}{2e^6}$ (B) $\frac{1}{e^6}$ (C) $\frac{2}{e^6}$ (D) $\frac{\pi}{2e^6}$ (E) infinite

12. Which of the following series converges for all real numbers x?

(A) $\sum_{n=1}^{\infty} \frac{x^n}{n}$ (B) $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$ (C) $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$ (D) $\sum_{n=1}^{\infty} \frac{e^n x^n}{n!}$ (E) $\sum_{n=1}^{\infty} \frac{n! x^n}{e^n}$

13. $\int_1^e \frac{x^2+1}{x}\,dx =$

(A) $\frac{e^2-1}{2}$ (B) $\frac{e^2+1}{2}$ (C) $\frac{e^2+2}{2}$ (D) $\frac{e^2-1}{e^2}$ (E) $\frac{2e^2-8e+6}{3e}$

x	0	1	2	3
$f''(x)$	5	0	–7	4

14. The polynomial function f has selected values of its second derivative f'' given in the table above. Which of the following statements must be true?

(A) f is increasing on the interval $(0, 2)$.

(B) f is decreasing on the interval $(0, 2)$.

(C) f has a local maximum at $x = 1$.

(D) The graph of f has a point of inflection at $x = 1$.

(E) The graph of f changes concavity in the interval $(0, 2)$.

15. If $f(x) = (\ln x)^2$, then $f''(\sqrt{e}) =$

(A) $\frac{1}{e}$ (B) $\frac{2}{e}$ (C) $\frac{1}{2\sqrt{e}}$ (D) $\frac{1}{\sqrt{e}}$ (E) $\frac{2}{\sqrt{e}}$

16. What are all values of x for which the series $\sum_{n=1}^{\infty}\left(\frac{2}{x^2+1}\right)^n$ converges?

(A) $-1 < x < 1$

(B) $x > 1$ only

(C) $x \geq 1$ only

(D) $x < -1$ and $x > 1$ only

(E) $x \leq -1$ and $x \geq 1$

17. Let h be a differentiable function, and let f be the function defined by $f(x) = h(x^2 - 3)$. Which of the following is equal to $f'(2)$?

(A) $h'(1)$ (B) $4h'(1)$ (C) $4h'(2)$ (D) $h'(4)$ (E) $4h'(4)$

18. In the xy-plane, the line $x+y=k$, where k is a constant, is tangent to the graph of $y=x^2+3x+1$. What is the value of k?

(A) -3 (B) -2 (C) -1 (D) 0 (E) 1

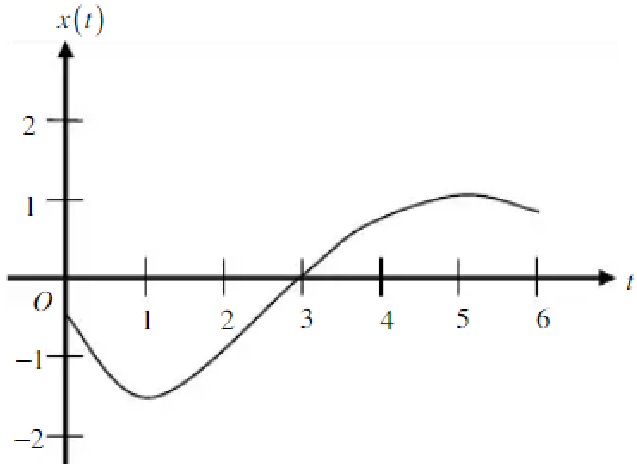
19. $\int \frac{7x}{(2x-3)(x+2)}\,dx =$ (A) $\frac{3}{2}\ln|2x-3|+2\ln|x+2|+C$

(B) $3\ln|2x-3|+2\ln|x+2|+C$ (C) $3\ln|2x-3|-2\ln|x+2|+C$

(D) $-\frac{6}{(2x-3)^2}-\frac{2}{(x+2)^2}+C$ (E) $-\frac{3}{(2x-3)^2}-\frac{2}{(x+2)^2}+C$

20. What is the sum of the series $1+\ln 2+\frac{(\ln 2)^2}{2!}+\cdots+\frac{(\ln 2)^n}{n!}+\cdots$?

(A) $\ln 2$ (B) $\ln(1+\ln 2)$ (C) 2 (D) e^2 (E) The series diverges.



21. A particle moves along a straight line. The graph of the particle's position $x(t)$ at time t is shown above for $0<t<6$. The graph has horizontal tangents at $t=1$ and $t=5$ and a point of inflection at $t=2$. For what values of t is the velocity of the particle increasing?

(A) $0<t<2$

(B) $1<t<5$

(C) $2<t<6$

(D) $3<t<5$ only

(E) $1<t<2$ and $5<t<6$

x	0	1
$f(x)$	2	4
$f'(x)$	6	–3
$g(x)$	–4	3
$g'(x)$	2	–1

22. The table above gives values of f, f', g and g' for selected values of x. If $\int_0^1 f'(x)g(x)dx = 5$, then $\int_0^1 f(x)g'(x)dx =$

23. If $f(x) = x\sin(2x)$, which of the following is the Taylor series for f about $x = 0$?

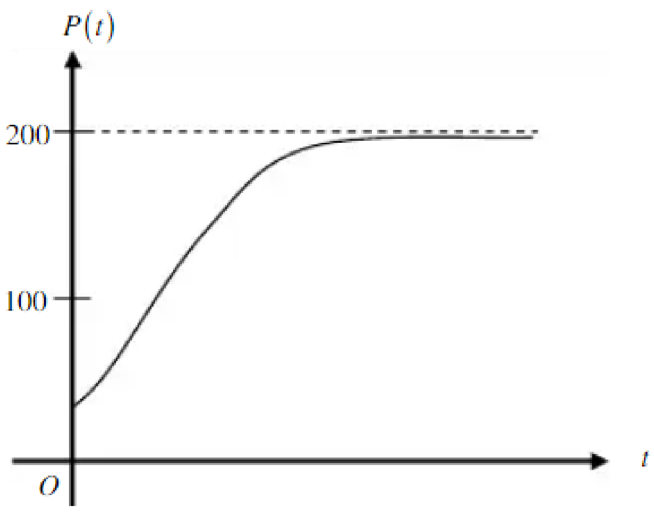
(A) $x - \frac{x^3}{2!} + \frac{x^5}{4!} - \frac{x^7}{6!} + \cdots$

(B) $x - \frac{4x^3}{2!} + \frac{16x^5}{4!} - \frac{64x^7}{6!} + \cdots$

(C) $2x - \frac{8x^3}{3!} + \frac{32x^5}{5!} - \frac{128x^7}{7!} + \cdots$

(D) $2x^2 - \frac{2x^4}{3!} + \frac{2x^6}{5!} - \frac{2x^8}{7!} + \cdots$

(E) $2x^2 - \frac{8x^4}{3!} + \frac{32x^6}{5!} - \frac{128x^8}{7!} + \cdots$



24. Which of the following differential equations for a population P could model the logistic growth shown in the figure above?

(A) $\frac{dP}{dt} = 0.2P - 0.001P^2$

(B) $\frac{dP}{dt} = 0.1P - 0.001P^2$

(C) $\frac{dP}{dt} = 0.2P^2 - 0.001P$

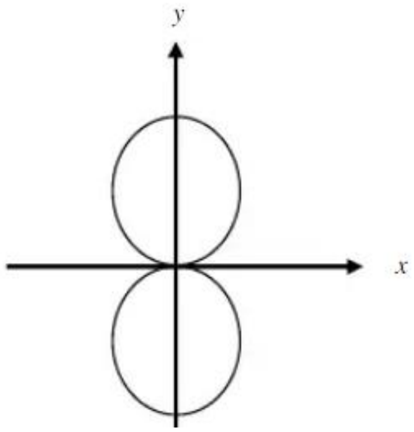
(D) $\frac{dP}{dt} = 0.1P^2 - 0.001P$

(E) $\frac{dP}{dt} = 0.1P^2 + 0.001P$

$$f(x) = \begin{cases} cx + d & \text{for } x \le 2 \\ x^2 - cx & \text{for } x > 2 \end{cases}$$

25. Let f be the function defined above, where c and d are constants. If f is differentiable at $x = 2$, what is the value of $c + d$?

(A) -4 (B) -2 (C) 0 (D) 2 (E) 4

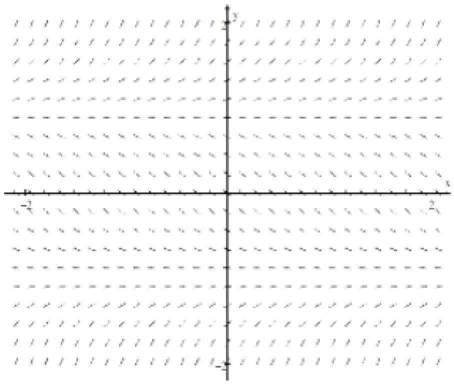


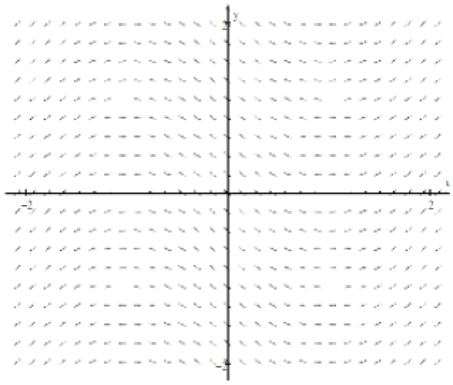
26. Which of the following expressions gives the total area enclosed by the polar curve $r = \sin^2\theta$ shown in the figure above?

(A) $\frac{1}{2}\int_0^{\pi} \sin^2\theta\, d\theta$ (B) $\int_0^{\pi} \sin^2\theta\, d\theta$ (C) $\frac{1}{2}\int_0^{\pi} \sin^4\theta\, d\theta$ (D) $\int_0^{\pi} \sin^4\theta\, d\theta$

(E) $2\int_0^{\pi} \sin^4\theta\, d\theta$

27. Which of the following could be the slope field for the differential equation $\frac{dy}{dx} = y^2 - 1$?


(A)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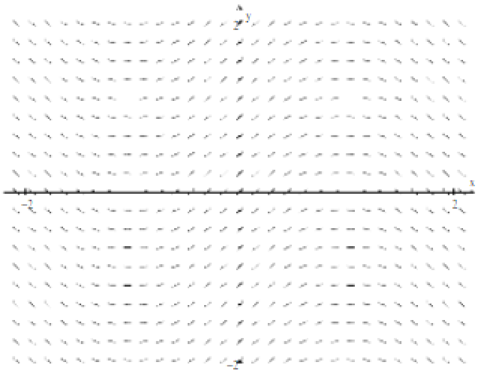
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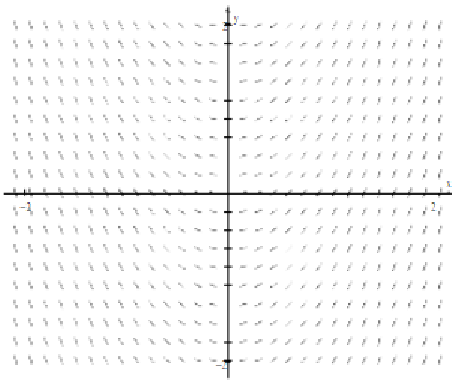
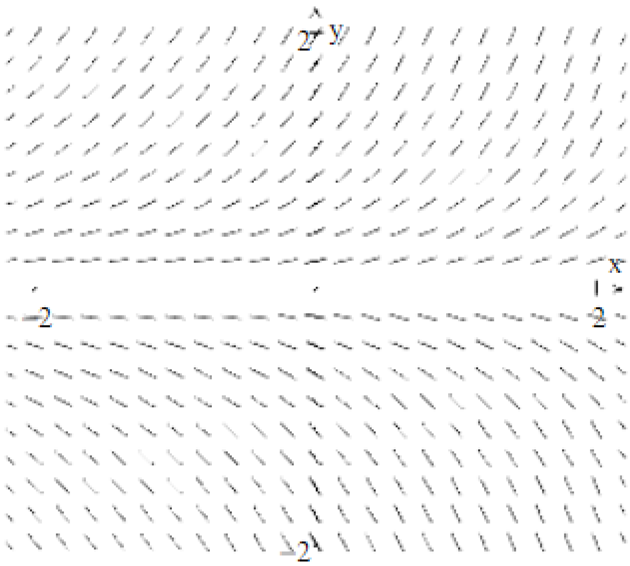
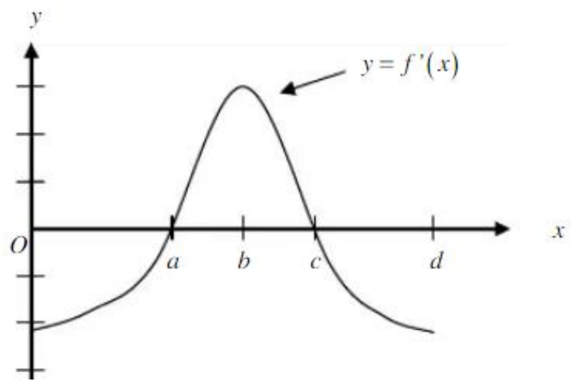

(D)


(E)

28. In the xy-plane, a particle moves along the parabola $y = x^2 - x$ with a constant speed of $2\sqrt{10}$ units per second. If $\frac{dx}{dt} > 0$, what is the value of $\frac{dy}{dt}$ when the particle is at the point $(2,2)$?

(A) $\frac{2}{3}$ (B) $\frac{2\sqrt{10}}{3}$ (C) 3 (D) 6 (E) $6\sqrt{10}$



76. The graph of f', the derivative of a function f, is shown above. The domain of f is the open interval $0 < x < d$. Which of the following statements is true?

(A) f has a local minimum at $x = c$.

(B) f has a local maximum at $x = b$.

(C) The graph of f has a point of inflection at $(a, f(a))$.

(D) The graph of f has a point of inflection at $(b, f(b))$.

(E) The graph of f is concave up on the open interval (c, d).

77. Water is pumped out of a lake at the rate $R(t) = 12\sqrt{\frac{t}{t+1}}$ cubic meters per minute, where t is measured in minutes. How much water is pumped from time $t = 0$ to $t = 5$?

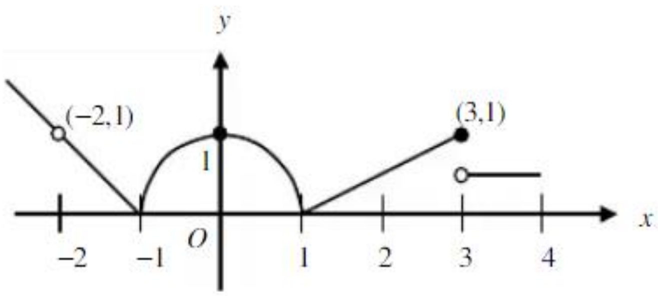
(A) 9.439 cubic meters

(B) 10.954 cubic meters

(C) 43.816 cubic meters

(D) 47.193 cubic meters

(E) 54.772 cubic meters



Graph of f

78. The graph of a function f is shown above. For which of the following values of c does $\lim_{x \to c} f(x) = 1$?

(A) 0 only

(B) 0 and 3 only

(C) −2 and 0 only

(D) −2 and 3 only

(E) −2, 0, and 3

79. Let f be a positive, continuous, decreasing function such that $a_n = f(n)$. If $\sum_{n=1}^{\infty} a_n$ converges to k, which of the following must be true?

(A) $\lim_{n \to \infty} a_n = k$

(B) $\int_1^n f(x)dx = k$

(C) $\int_1^{\infty} f(x)dx$ diverges.

(D) $\int_1^{\infty} f(x)dx$ converges.

(E) $\int_1^{\infty} f(x)dx = k$

80. The derivative of the function f is given by $f'(x) = x^2 \cos(x^2)$. How many points of inflection does the graph of f have on the open interval $(-2, 2)$?

(A) One (B) Two (C) Three (D) Four (E) Five

81. Let f and g be continuous functions for $a \le x \le b$. If $a < c < b$, $\int_a^b f(x)dx = P$, $\int_c^b f(x)dx = Q$, $\int_a^b g(x)dx = R$, and $\int_c^b g(x)dx = S$, then $\int_a^c (f(x) - g(x))dx = ?$

(A)P-Q+R-S (B)P-Q-R+S (C)P-Q-R-S (D)P+Q-R-S (E)P+Q-R+S

82. If $\sum_{n=1}^{\infty} a_n$ diverges and $0 \le a_n \le b_n$ for all n, which of the following statements must be true?

(A) $\sum_{n=1}^{\infty} (-1)^n a_n$ converges. (B) $\sum_{n=1}^{\infty} (-1)^n b_n$ converges. (C) $\sum_{n=1}^{\infty} (-1)^n b_n$ diverges.

(D) $\sum_{n=1}^{\infty} b_n$ converges. (E) $\sum b_n$ diverges.

83. What is the area enclosed by the curves $y = x^3 - 8x^2 + 18x - 5$ and $y = x + 5$?

(A)10.667 (B) 11.833 (C) 14.583 (D) 21.333 (E) 32

84. Let f be a function with $f(3)=2$, $f'(3)=-1$, $f''(3)=6$, and $f'''(3)=12$. Which of the following is the third-degree Taylor polynomial for f about $x=3$?

(A) $2-(x-3)+3(x-3)^2+2(x-3)^3$

(B) $2-(x-3)+3(x-3)^2+4(x-3)^3$

(C) $2-(x-3)+6(x-3)^2+12(x-3)^3$

(D) $2-x+3x^2+2x^3$

(E) $2-x+6x^2+12x^3$

85. A particle moves on the x-axis with velocity given by $v(t)=3t^4-11t^2+9t-2$ for $-3\le t\le 3$. How many times does the particle change direction as t increases from -3 to 3?

(A) Zero (B) One (C) Two (D) Three (E) Four

86. On the graph of $y=f(x)$, the slope at any point (x, y) is twice the value of x. If $f(2)=3$, what is the value of $f(3)$?

(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

87. An object traveling in a straight line has position $x(t)$ at time t. If the initial position is $x(0)=2$ and the velocity of the object is $v(t)=\sqrt[3]{1+t^2}$, what is the position of the object at time $t=3$?

(A) 0.431 (B) 2.154 (C) 4.512 (D) 6.512 (E) 17.408

88. For all values of x, the continuous function f is positive and decreasing. Let g be the function given by $g(x)=\int_2^x f(t)\,dt$. Which of the following could be a table of values for g?

(A)

x	$g(x)$
1	−2
2	0
3	1

(B)

x	$g(x)$
1	−2
2	0
3	3

(C)

x	$g(x)$
1	1
2	0
3	−2

(D)

x	$g(x)$
1	2
2	0
3	−1

(E)

x	$g(x)$
1	3
2	0
3	2

89. The function f is continuous for $-2 \le x \le 2$ and $f(-2) = f(2) = 0$. If there is no c, where $-2 < c < 2$, for which $f'(c) = 0$, which of the following statements must be true?

(A) For $-2 < k < 2$, $f'(k) > 0$.

(B) For $-2 < k < 2$, $f'(k) < 0$.

(C) For $-2 < k < 2$, $f'(k)$ exists.

(D) For $-2 < k < 2$, $f'(k)$ exists, but f' is not continuous.

(E) For some k, where $-2 < k < 2$, $f'(k)$ does not exist.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-1	-5	1	3	0
0	-2	0	1	1
1	0	-3	0	0.5
2	5	-1	5	2

90. The table above gives values of the differentiable functions f and g and of their derivatives f' and g', at selected values of x. If $h(x) = f(g(x))$, what is the slope of the graph of h at $x = 2$?

(A) -10 (B) -6 (C) 5 (D) 6 (E) 10

91. Let f be the function given by $f(x) = \int_{1/3}^{x} \cos\left(\frac{1}{t^2}\right) dt$ for $\frac{1}{3} \le x \le 1$. At which of the following values of x does f attain a relative maximum?

(A) 0.357 and 0.798 (B) 0.4 and 0.564 (C) 0.4 only (D) 0.461 (E) 0.999