APCalculusTest01

1. The function f is given by $f(x) = 3x^4 - 2x^3 + 7x - 2$. On which of the following intervals is f' decreasing?

(A) $(-\infty,\infty)$

(B) $(-\infty,0)$

(C) $\left(\frac{1}{3},\infty\right)$

(D) $\left(0,\frac{1}{3}\right)$

(E) $\left(-\frac{1}{3},0\right)$

2. What is the area under the curve described by the parametric equations $x = \sin t$ and $y = \cos^2 t$ for $0 \le t \le \frac{\pi}{2}$?

(A) $\frac{1}{3}$

(B) $\frac{1}{2}$

(C) $\frac{2}{3}$

(D) 1

(E) $\frac{4}{3}$

3. The function f is given by $f(x) = 8x^3 + 36x^2 + 54x + 27$. All of these statements are true EXCEPT

(A) $-\frac{3}{2}$ is a zero of f.

(B) $-\frac{3}{2}$ is a point of inflection of f.

(C) $-\frac{3}{2}$ is a local extremum of f.

(D) $-\frac{3}{2}$ is a zero of the derivative of f.

(E) f is strictly monotonic.

4. $\int x \ln x \, dx =$

(A) $\frac{x^2 \ln x}{2} + \frac{x^2}{4} + C$

(B) $\frac{x^2}{4}(2 \ln x - 1) + C$

(C) $\frac{x}{2}(x \ln x - 2 + C)$

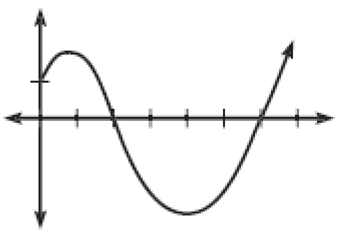
(D) $x \ln x - \frac{x^2}{4} + C$

(E) $\frac{(\ln x)}{x} - \frac{x^2}{4} + C$

5. Let $h(x) = \ln|g(x)|$. If g is decreasing for all x in its domain, then

(A) h is strictly increasing.
(B) h is strictly decreasing.
(C) h has no relative extrema.
(D) both (B) and (C).
(E) none of the above.

QUESTIONS 6, 7, AND 8 REFER TO THE DIAGRAM AND INFORMATION BELOW.



The function f is defined on [0,7]. The graph of its derivative, f', is shown above.

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6. The point (2,5) is on the graph of $y = f(x)$. An equation of the line tangent to the graph of f at (2,5) is

(A)y=2 (B)y=5 (C)y=0 (D)y=2x+5 (E)y=2x-5

7.How many points of inflection does the graph y=f(x) have over [0,7] ?
(A)0 (B)1 (C)2 (D)3 (E)4

8. At what value of x does the absolute maximum value of f occur?
(A)1 (B)2 (C)4 (D)6 (E)7

9. $\int_1^e \left(\frac{x^2+4}{x}\right)dx =$

(A) $\frac{e^2+9}{2}$

(B) $\frac{e^2-9}{2}$

(C) $\frac{e^2+7}{2}$

(D) $\frac{e^2+8}{2}$

(E) $\frac{e^2-4}{2}$

10. The function f given by $f(x) = 3x^5 - 4x^3 - 3x$ is increasing and concave up over which of these intervals?

(A) $\left(-\infty, -\sqrt{\frac{2}{5}}\right)$

(B) $\left(-\sqrt{\frac{2}{5}}, 0\right)$

(C) $(-1, 1)$

(D) $\left(\sqrt{\frac{2}{5}}, \infty\right)$

(E) $(1, \infty)$

11. If $y=2xy-x^2+3$，then when x=1，$\frac{dy}{dx}=$

(A)-6 (B)-2 (C)$-\frac{2}{3}$ (D)2 (E)6

12. The length of the curve described by the parametric equations $x = 2t^3$ and $y = t^3$ where $0 \le t \le 1$ is

(A) $\frac{5}{7}$

(B) $\frac{\sqrt{5}}{2}$

(C) $\frac{3}{2}$

(D) $\sqrt{5}$

(E) 3

13. What is the average value of $f(x) = 3\sin^2 x - \cos^2 x$ over $\left[0, \frac{\pi}{2}\right]$?

(A)0 (B)1 (C)$\sqrt{2}$ (D)$\sqrt{3}$ (E)$\frac{\pi}{2}$

14. Let f be defined as

$$f(x) = \begin{cases} \sqrt[3]{x} + kx, & x < 1 \\ \ln x, & x \ge 1 \end{cases}$$

for some constant k. For what value of k will f be differentiable over its whole domain?

(A)-2 (B)-1 (C)$\frac{2}{3}$ (D)1 (E)None of the above

15. What is the approximation of the value of e^3 obtained by using a fourth-degree Taylor polynomial about $x = 0$ for e^x?

(A) $1+3+\frac{9}{2}+\frac{9}{2}+\frac{27}{8}$

(B) $1+3+9+\frac{27}{8}$

(C) $1+3+\frac{27}{8}$

(D) $3-\frac{9}{2}+\frac{9}{2}-\frac{27}{4}$

(E) $3+9+\frac{27}{8}$

16. $\int 6x^3e^{3x}dx =$

(A) $e^{3x}(9x^3 - 9x^2 + 6x - 2) + C$

(B) $e^{3x}\left(2x^3-2x^2-\frac{4}{3}x+\frac{4}{9}\right)+C$

(C) $\frac{2}{9}e^{3x}\left(2x^3-2x^2+\frac{4}{3}x-\frac{4}{9}\right)+C$

(D) $\frac{2}{9}e^{3x}\left(9x^3-9x^2-6x-2\right)+C$

(E) $\frac{2}{9}e^{3x}\left(9x^3-9x^2+6x-2\right)+C$

17. If $f(x) = \sec x$, then $f'(x)$ has how many zeros over the closed interval $[0,2\pi]$?

(A)0 (B)1 (C)2 (D)3 (E)4

18. Consider the region in the first quadrant bounded by $y = x^2$ over [0,3]. Let L_3 represent the Riemann approximation of the area of this region using left endpoints and three rectangles, R_3 represent the Riemann approximation using right endpoints and three rectangles, M_3 represent the Riemann approximation using midpoints and three rectangles, and T_3 represent the trapezoidal approximation with three trapezoids. Which of the following statements is true?

(A) $R_3 < T_3 < \int_0^3 x^2 dx < M_3 < L_3$

(B) $L_3 < M_3 < T_3 < R_3 < \int_0^3 x^2 dx$

(C) $M_3 < L_3 < \int_0^3 x^2 dx < T_3 < R_3$

(D) $L_3 < M_3 < \int_0^3 x^2 dx < R_3 < T_3$

(E) $L_3 < M_3 < \int_0^3 x^2 dx < T_3 < R_3$

19. Which of the following series converge?

I. $\sum_{n=1}^{\infty}\left(\frac{2^n}{n+1}\right)$

II. $\sum_{n=1}^{\infty}\frac{3}{n}$

III. $\sum_{n=1}^{\infty}\left(\frac{\cos 2n\pi}{n^2}\right)$

(A) I only
(B) II only
(C) III only
(D) I and II
(E) I and III

20. The area of the region inside the polar curve $r = 4\sin\theta$ but outside the polar curve $r = 2\sqrt{2}$ is given by

(A) $2\int_{\pi/4}^{3\pi/4}\left(4\sin^2\theta - 1\right)d\theta$

(B) $\frac{1}{2}\int_{\pi/4}^{3\pi/4}\left(4\sin\theta - 2\sqrt{2}\right)^2 d\theta$

(C) $\frac{1}{2}\int_{\pi/4}^{3\pi/4}\left(4\sin\theta - 2\sqrt{2}\right)d\theta$

(D) $\frac{1}{2}\int_{\pi/4}^{3\pi/4}\left(16\sin^2\theta - 8\right)d\theta$

(E) $\frac{1}{2}\int_{\pi/4}^{3\pi/4}\left(4\sin^2\theta - 1\right)d\theta$

21. When $x = 16$, the rate at which $x^{3/4}$ is increasing is k times the rate at which $\sqrt{x}$ is increasing. What is the value of k?

(A) $\frac{1}{8}$ (B) $\frac{3}{8}$ (C)2 (D)3 (E)8

22. The length of the path described by the parametric equations $x = 2\cos 2t$ and $y = \sin^2 t$ for $0 \le t \le \pi$ is given by

(A) $\int_0^{\pi}\sqrt{4\cos^2 2t + \sin^4 t}\ dt$

(B) $\int_0^{\pi}\sqrt{2\sin t\cos t - 4\sin 2t}\ dt$

(C) $\int_0^{\pi}\sqrt{4\sin^2 t\cos^2 t - 16\sin^2 2t}\ dt$

(D) $\int_0^{\pi}\sqrt{4\sin^2 2t + 4\sin^2 t\cos^2 t}\ dt$

(E) $\int_0^{\pi}\sqrt{16\sin^2 2t + 4\sin^2 t\cos^2 t}\ dt$

23. Determine the interval of convergence for the series $\sum_{n=0}^{\infty}\left(\frac{(3x-2)^{n+2}}{n^{5/2}}\right)$.

(A) $-\frac{1}{3} \le x \le \frac{1}{3}$

(B) $-\frac{1}{3} < x < 1$

(C) $-\frac{1}{3} \le x \le 1$

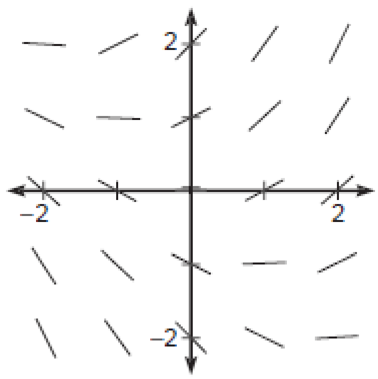
(D) $\frac{1}{3} \le x \le 1$

(E) $-\frac{1}{3} \le x \le -1$

24. $f(x)=\frac{(3x+4)(2x-1)}{(2x-3)(2x+1)}$ has a horizontal asymptote at $x =$

(A) $\frac{3}{2}$ (B) $\frac{3}{2}$ and $-\frac{1}{2}$ (C)0 (D) $-\frac{3}{4}$ and $\frac{1}{2}$ (E)None of the above

25.



Shown above is the slope field for which of the following differential equations?

(A) $\frac{dy}{dx}=1+x$ (B) $\frac{dy}{dx}=x-y$ (C) $\frac{dy}{dx}=\frac{x+y}{2}$ (D) $\frac{dy}{dx}=y-x$ (E) $\frac{dy}{dx}=y+1$

26. $\int_{2}^{\infty}\frac{x^2}{e^x}dx =$

(A) $\frac{5}{e}$ (B) $10e^2$ (C) $\frac{10}{e^2}$ (D)2 (E)5e

27. The population $P(t)$ of a species satisfies the logistic differential equation $\frac{dP}{dt} = \frac{2}{3}P\left(5 - \frac{P}{100}\right)$. What is $\lim_{t\to\infty} P(t)$?

(A)100 (B)200 (C)300 (D)400 (E)500

28. If $\sum_{n=0}^{\infty} a_n(x-c)^n$ is a Taylor series that converges to $f(x)$ for every real x, then $f''(c) =$

(A) 0

(B) $n(n-1)a_n$

(C) $\sum_{n=0}^{\infty} na_n(x-c)^{n-1}$

(D) $\sum_{n=0}^{\infty} a_n$

(E) $\sum_{n=0}^{\infty} n(n-1)a_n(x-c)^{n-2}$

29. The graph of the function represented by the Taylor series, centered at $x = 1$, $1 - (x-1) + (x-1)^2 - (x-1)^3 + \ldots = (-1)^n(x-1)^n$ intersects the graph of $y = e^x$ at $x =$

(A)-9.425 (B)0.567 (C)0.703 (D)0.773 (E)1.763

30. If f is a vector-valued function defined by $f(t) = \langle\cos^2 t, \ln t\rangle$, then $f''(t) =$

(A) $\left\langle -2\cos t \sin t, \frac{1}{t}\right\rangle$

(B) $\left\langle 2\cos t, \frac{1}{t}\right\rangle$

(C) $\left\langle 2\cos t \ \sin t, \frac{1}{t}\right\rangle$

(D) $\left\langle -2\cos^2 t + 2\sin^2 t, -\frac{1}{t^2}\right\rangle$

(E) $\left\langle -2, -\frac{1}{t^2}\right\rangle$

31. The diagonal of a square is increasing at a constant rate of $\sqrt{2}$ centimeters per second. In terms of the perimeter, P, what is the rate of change of the area of the square in square centimeters per second?

(A) $\frac{\sqrt{2}P}{4}$ (B) $\frac{4P}{\sqrt{2}}$ (C)2P (D)P (E) $\frac{P}{2}$

32. If f is continuous over the set of real numbers and f is defined as $f(x)=\frac{x^2-3x+2}{x-2}$ for all $x \neq 2$, then $f(2)$ =

(A)-2 (B)-1 (C)0 (D)1 (E)2

33. If $0 \leq k \leq 2$ and the area between the curves $y = x^2 + 4$ and $y = x^3$ from $x = 0$ to $x = k$ is 5, then k =

(A)1.239 (B)1.142 (C)1.029 (D)0.941 (E)0.876

34. Determine $\frac{dy}{dx}$ for the curve defined by $x\sin y = 1$.

(A) $-\frac{\tan y}{x}$

(B) $\frac{\tan y}{x}$

(C) $\frac{\sec y - \tan y}{x}$

(D) $\frac{\sec y}{x}$

(E) $-\frac{\sec y}{x}$

35. If $f(x) = h(x) + g(x)$ for $0 \le x \le 10$,
then $\int_0^{10} \left(f(x) - 2h(x) + 3\right) dx =$

(A) $2\int_0^{10} \left(g(x) - h(x) + 3\right) dx$ (B)g(10)-h(10)+30 (C)g(10)-h(10)+30-g(0)-h(0)

(D) $\int_0^{10} \left(g(x) - h(x)\right) dx + 30$ (E) $\int_0^{10} \left(g(x) - 2h(x)\right) dx + 30$

36. Use a fifth-degree Taylor polynomial centered at $x = 0$ to estimate e^2.

(A)7.000 (B)7.267 (C)7.356 (D)7.389 (E)7.667

37. What are all the values of x for which the series $\sum_{n=1}^{\infty} \left(\frac{(x+2)^n}{\left(n\sqrt{n}\,3^n\right)} \right)$ converges?

(A) $-3 < x < 3$
(B) $-3 \le x \le 3$
(C) $-5 < x < 1$
(D) $-5 \le x \le 1$
(E) $-5 \le x < 1$

38. Let $f(x) = \left| x^2 - 4 \right|$. Let R be the region bounded by f, the x-axis, and the vertical lines $x = -3$ and $x = 3$. Let T_6 represent the approximation of the area of R using the trapezoidal rule with $n = 6$. The quotient
$\frac{T_6}{\int_{-3}^{3} f(x)dx} =$

(A)0.334 (B)0.978 (C)1.022 (D)1.304 (E)4.666

39. Let R be the region bounded by $y = 3 - x^2$, $y = x^3 + 1$, and $x = 0$. If R is rotated about the x-axis, the volume of the solid formed could be determined by

(A) $\pi\int_0^1\left(\left(x^3+1\right)^2-\left(3-x^2\right)^2\right)dx$

(B) $-\pi\int_1^0\left(\left(x^3+1\right)^2-\left(3-x^2\right)^2\right)dx$

(C) $2\pi\int_0^1\left(x\left(-x^3-x^2+2\right)\right)dx$

(D) $\pi\int_1^0\left(\left(x^3+1\right)^2-\left(3-x^2\right)^2\right)dx$

(E) $2\pi\int_0^1\left(x\left(x^3+x^2-2\right)\right)dx$

40. Let f be defined as

$$f(x) = \begin{cases} -x^2, & x \le 0 \\ \sqrt{x}, & x > 0 \end{cases}$$

and g be defined as

$g(x) = \int_{-4}^{x} f(t)dt$ for $-4 \le t \le 4$.

Which of these is an equation for the tangent line to g at $x = 2$?

(A) $4x + 3y = 4\sqrt{2} + 72$

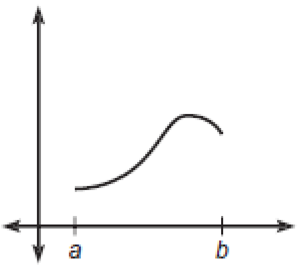
(B) $3x\sqrt{2} - 3y = -64 - 2\sqrt{2}$

(C) $3x\sqrt{2} - 3y = 64 - 2\sqrt{2}$

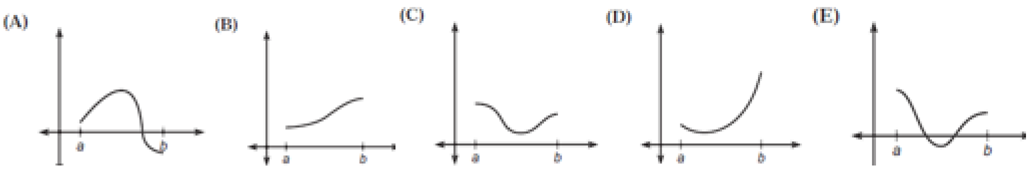
(D) $3x\sqrt{2} - 3y = 64 + 2\sqrt{2}$

(E) $4x + 3y = 4\sqrt{2} - 56$

41.



Let $g(x) = \int_a^x f(t)dt$ *t*, where $a \le x \le b$. The figure above shows the graph of g on $[a,b]$. Which of the following could be the graph of f on $[a,b]$?



42. The sum of the infinite geometric series $\frac{4}{5} + \frac{8}{35} + \frac{16}{245} + \frac{32}{1715} + \ldots$ is

(A) 0.622
(B) 0.893
(C) 1.120
(D) 1.429
(E) 2.800

43. Let f be a strictly monotonic differentiable function on the closed interval [5,10] such that $f(5) = 6$ and $f(10) = 26$. Which of the following must be true for the function f on the interval [5,10]?

I. The average rate of change of f is 4.

II. The absolute maximum value of f is 26.

III. $f'(8) > 0$.

(A) I only
(B) II only
(C) III only
(D) I and II
(E) I, II, and III

44. Let $F(x)$ be an antiderivative of $f(x) = e^{2x}$. If $F(0) = 2.5$, then $F(5) =$

(A) 150.413
(B) 11013.233
(C) 11015.233
(D) 22026.466
(E) 22028.466

45. The base of a solid is the region in the first quadrant bounded by $y = -x^2 + 3$. The cross sections perpendicular to the x-axis are squares. Find the volume of the solid.

(A) 3.464
(B) 8.314
(C) 8.321
(D) 16.628
(E) 21.600

1. D	7. C	13. B	19. C	24. A
2. C	8. B	14. E	20. D	25. C
3. C	9. C	15. A	21. D	26. C
4. B	10. E	16. E	22. E	27. E
5. C	11. E	17. D	23. D	28. A
6. B	12. D	18. E		

29. B	33. A	37. D	40. D	43. E
30. D	34. A	38. B	41. A	44. C
31. E	35. D	39. D	42. C	45. B
32. D	36. B			

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